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CP Violation: a Basic Introduction*

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ABSTRACT

We present a simple introduction to CP violation, stressing the importance of CP as a natural symmetry of the gauge interactions, broken by scalar couplings. In section 5, named "CP against TCP" a particular treatment of the K system relates ϵ' to the notion of explicit CP violation showing the crucial rôle of strong-interaction phases, in close parallel to the situation in B mesons. For the sake of comparison, some notions about Left–Right-symmetric models are introduced; their contributions to other channels of CP violation are contrasted with those of the standard model.

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These notes aim at presenting the basic aspects of CP violation. In a first part, I will as much as possible try to give a simple, nontechnical vision of this subject. Later, I will introduce an alternative approach, based on $SU(2)_L \times SU(2)_R \times U(1)$],in order to study to what extent theoretical predictions can depart from the standard model expectations.

1. INTRODUCTION

1.1. CP Violation – Principles

The notion of CP violation is in principle very straightforward and can be formulated as a simple recipe: consider a real experiment, examine its image in a mirror, and build an apparatus identical to the reflected image. Furthermore, make sure that this apparatus is built exclusively from antimatter (!). Comparing the initial experiment (particle trajectories, branching ratios) to the one performed with this CP-conjugate apparatus, any discrepancy constitutes an instance of CP violation.

The only drawback of this description is ... that the experiment is impossible to realize practically, at least for any macroscopic piece of equipment.

This remark is indeed quite general when discrete symmetries are considered: if one wishes to describe any experiment completely, including the measuring devices (and even the observer), the symmetric experiment (be it the P-, C-, or T-conjugate) is in practice impossible to build.

On the contrary, when things are considered at the level of a small number of elementary particles in interaction, it is relatively easy to build up the relevant situations. Of course the situation is still globally asymmetric (since the distant apparatus is always constituted of matter); however, the comparison of the size (or rather the disproportion) of the macroscopic and microscopic energies involved, or of the different time scales implied, leads to the conclusion that any asymmetry observed at the microscopic level is indeed inherent in the system studied.

Even if we immediately accept this proviso, we must admit that the most usual description of CP violation ("the longer-lived neutral K meson occasionally decays to two pions") is far from transparent, and that a considerable amount of decyphering is needed to relate this formulation to our starting point.

A simple situation can however be extracted from the study of the K^0 system, and presented in terms close to those used before (see fig 1.1). In fact, even if the full experiment is not directly feasible in the laboratory, we know enough about its various parts to predict the result with total confidence.

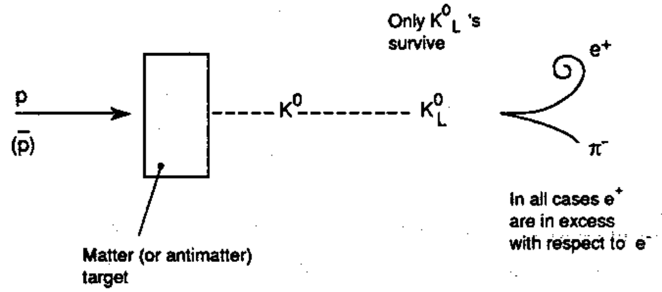


Figure 1.1

Let us indeed consider a beam of energetic particles (protons, pions) hitting a matter target. Given enough energy, K^0's are produced. Far enough from the target only K_L^0's survive;looking at their semileptonic decays, we pay special attention to the ratio of the production rates for $(\nu_e e^+ \pi^-)$ and $(\bar{\nu}_e e^- \pi^+)$. The former are in slight excess:

$$R = \frac{\sigma(p + \text{cible} \to K_L^0 \to \nu_e e^+ \pi^-)}{\sigma(p + \text{cible} \to K_L^0 \to \bar{\nu}_e e^- \pi^0)} \equiv 1 + 4\text{Re}(\epsilon) \simeq 1.01 \quad (1.1)$$

This far, nothing in this observation calls for CP violation: since the initial state is not a CP eigenstate, there is no reason for the final state to be! We can however imagine that the experiment is repeated using both a beam and a target made of antimatter. Here again, K^0's are produced, and the same K_L^0 combination stands out because of its exceptional time of flight before decay. As a result, *the same excess of positrons with respect to electrons is observed in the final state!*

Whether we start from matter or antimatter, in all cases positrons dominate electrons far enough from the target. Clearly, this process does not respect charge conjugation (C) since we can define the positron in an absolute way (that is without

reference to the electron)! This difference in the numbers of positrons and electrons also violates CP, since when evaluating the production ratio, we have summed over all polarizations and all orientations of the momenta.

1.2. T VIOLATION

Lagrangian theories predict in very general terms the invariance of physical phenomena under the triple conjugation T, C, P. To any CP violation is thus associated a violation of T (the "time reversal"). We stress that we deal here only with microscopic phenomena.

A more familiar type of time-reversal violation is present at the macroscopic level; it is at the root of the second principle of thermodynamics with the irreversible increase of entropy. Such notions, including the ageing of living beings, are essentially macroscopic, and statistical mechanics describe how such phenomena arise from reversible microscopic interactions, at the classical or quantum level.

The question we are considering here addresses a more fundamental level, since we have now to face the lack of reversibility of microscopic processes themselves, e.g. the motion of a single particle.

In classical mechanics, the motion of a material point is thus invariant under T (time reversal). In other words, if a film is taken of some phenomenon, (for instance the motion of a pendulum or the fall of a ball) and projected backwards, an independent observer will be incapable to detect the inversion on the basis of classical mechanics alone: both the original motion and its time-reversed copy are equally possible. (The same is not true if we consider more complex phenomena, such as pouring a liquid, but we enter there in the already discarded realm of statistical mechanics.)

I found it amusing to give as a second example (see fig. 2) the trajectory followed by a neutron in some experimental set-up involving electrostatic and magnetic fields (both inhomogeneous) if we assume that the particle possesses an Electric Dipole Moment (E.D.M.) in addition to its well-known magnetic dipole. (no such E.D.M. has been observed to date, but it is an active and promising direction for the search of T violation).

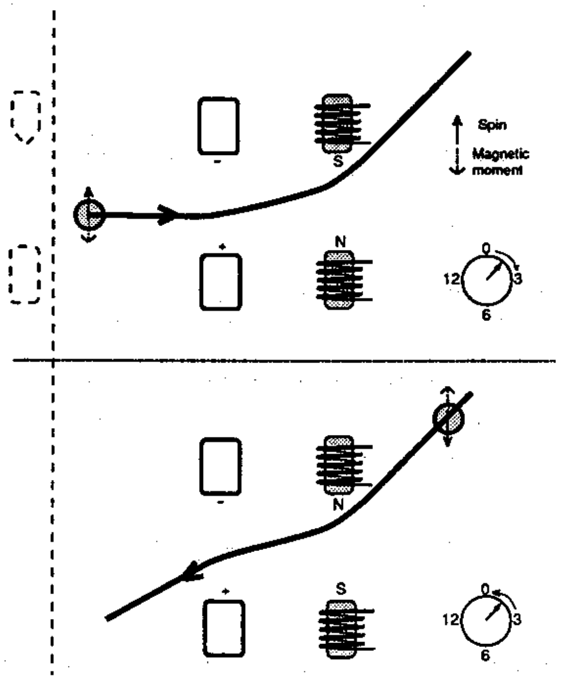


Fig 1.2

Maybe a word of commentary would be useful: the figure on top shows the expected motion, with the "usual" orientation of time. The spin of the neutron is drawn (dotted line) opposite to the neutron magnetic moment; to the left of the drawing, the profile of the polar pieces reminds us that an inhomogeneous field is necessary. The figure below shows the trajectory if the direction of t is reversed, with the obvious implication that the electric current, hence the magnetic field and the spin, flip. The presence of the T-violating E.D.M. results in different trajectories. From this experiment, it would thus be possible to determine whether a movie is projected normally or backwards!

2. GAUGE INTERACTIONS AND CP CONSERVATION

In this chapter, we attempt to show that CP is the natural symmetry for fermions in gauge theories.

We keep to (relatively) standard notations, and follow the ones adopted in the course by Landau and Lifschitz (Relativistic Quantum Mechanics); we also use their normalizations.

The obvious choice to study this problem would be to use 2-component semi-spinors. We will however stick to the habit of using 4-component Dirac spinors, while using the L and R projectors, referring to Left-handed and Right-handed fermions, respectively.

This helicity representation proves useful in a large number of reactions or decays mediated by weak interactions, as soon as the fermion masses can be neglected with respect to their energy. In the massless limit, an electron for instance appears as a mere superposition of two distinct and non-interacting particles (e_L and e_R).

It is worth insisting on this vision of things, according to which e_L and e_R are in fact two different particles, which we could as well call f_{1L} and f_{2R}, and which mostly ignore each other. Their association only originates from their interaction with scalar fields. As a reminder, we write down the Dirac equation for a 4-spinor Ψ of mass m:

$$(\not{p} - m)\Psi = 0, \tag{2.1}$$

with

$$\not{p} = p^\mu \gamma_\mu. \tag{2.2}$$

Using :

$$\begin{aligned} &L = \frac{1-\gamma_5}{2},\ R = \frac{1+\gamma_5}{2},\ R^2 = R,\ L^2 = L,\ R + L = 1,\ RL = LR = 0 \\ &\Psi = L\Psi + R\Psi = \frac{1-\gamma_5}{2}\Psi + \frac{1+\gamma_5}{2}\Psi \equiv \Psi_L + \Psi_R \end{aligned} \tag{2.3}$$

and multiplying successively to the left by L or R, we get (we take into account that

$\{\gamma_5, \gamma_\mu\} = 0, \gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3)$

$$\begin{aligned} L(\not p - m)\Psi = 0 = \not p R\Psi - mL\Psi = 0 \\ \not p \Psi_R - m\Psi_L = 0 \\ \not p \Psi_L - m\Psi_R = 0 \end{aligned} \quad (2.4)$$

These equations would decouple, were it not for the presence of the mass m, which links Ψ_L to Ψ_R.

We have this far not established any explicit link between the projectors L and R defined above and the particle's helicity. This link is obvious from the expression of the spin operator:

$$\frac{1}{2}\Sigma^\mu = \frac{1}{2}\gamma^0\gamma^5\gamma^\mu. \quad (2.5)$$

Its projection on the direction of motion is obtained by contracting with a unit vector defined as: $(0, p^i/|\vec{p}|)$, where $|\vec{p}| = |p^0|$ for $m = 0$. The Dirac (Weyl) equation for a massless particle Ψ_L then reduces to

$$\vec{\Sigma}.\vec{p}\, \frac{1-\gamma_5}{2}\Psi = -sign(p^0).\frac{1-\gamma_5}{2}\Psi \ . \quad (2.6)$$

For positive energy solutions, the spin is thus opposed to the direction of motion (left-handed or negative helicity), while for negative-energy solutions (to be reinterpreted as antiparticles) the helicity is positive (i.e. right-handed).

Let us pay a little more attention to the introduction of antiparticles. We expand Ψ as a superposition of plane waves, and write (λ_- stands for negative helicity, $\omega = |p^0|$):

$$\Psi_L(x,t) = \sum_{\substack{p^0>0 \\ \vec{p}}} a_{p^0,\vec{p},\lambda_-}\frac{e^{-ipx}}{\sqrt{2\omega}}u(\omega,\vec{p},\lambda_-) + \sum_{\substack{p^0<0 \\ \vec{p}}} a_{p^0,\vec{p},\lambda_+}\frac{e^{-ipx}}{\sqrt{2\omega}}u(-\omega,\vec{p},\lambda_+). \quad (2.7)$$

In order to avoid negative energies, we reinterpret the destruction of a negative-energy particle as the creation of a positive-energy antiparticle (in such a way, the energy balance is conserved).

By the same argument, the momentum balance requires that the antiparticle carries a momentum $\vec{p}$ opposite to that of the destroyed particle, and the same situation obtains for the spin $\vec{\sigma}$. The helicity $(\vec{p}.\vec{\sigma})/|\vec{p}|$ however, is obvioulsy left unchanged! Depending on the choice of spin $\vec{\sigma}$ or helicity, we thus write, defining the antiparticle creation operator b^+,

$$b^+_{\omega,\vec{p},\vec{\sigma}} = a_{-\omega,-\vec{p},-\vec{\sigma}}. \quad (2.8)$$

Upon the introduction of $v(\omega,\vec{p},\lambda_+) \equiv u(-\omega,-\vec{p},\lambda_+)$, we get

$$\Psi_L(x,t) = \sum_{p^0=\omega,\vec{p}} \{a_{\omega,\vec{p},\lambda_-}\frac{e^{-ipx}}{\sqrt{2\omega}}u(\omega,\vec{p},\lambda_-) + b^+_{\omega,\vec{p},\lambda_+}\frac{e^{ipx}}{\sqrt{2\omega}}v(\omega,\vec{p},\lambda_+)\} \quad (2.9)$$

Once this notation is established, we proceed with the observation that gauge interactions respect chirality. This affirmation is strictly valid in the framework of a broken symmetry only to the extent that we do not constrain the fermions to be on their "mass shell" , that is we don't impose that they satisfy the massive Dirac equation, which , as we have already seen, mixes spinors of different chiralities. Using $\Psi_1 = \Psi_{1L} + \Psi_{1R}$ and a similar decomposition for Ψ_2, we consider the vector

$$V^\mu = \bar{\Psi}_1\gamma^\mu\Psi_2, \quad (2.10)$$

which reduces to

$$V^\mu = \bar{\Psi}_{1L}\gamma^\mu\Psi_{2L} + \bar{\Psi}_{1R}\gamma^\mu\Psi_{2R}. \quad (2.11)$$

(Depending on the precise interaction considered. we may or may not identify the spinors Ψ_1 and Ψ_2.)

Observe that $\bar{\Psi}_{1L}$ corresponds here to the conjugate of Ψ_{1L}, i.e.

$$\bar{\Psi}_{1L} \equiv \overline{(\Psi_{1L})} = \left(\frac{1-\gamma_5}{2}\Psi_1\right)^+ \gamma_0 \quad (2.12)$$

On the contrary, scalar interactions mix the left- and right-handed chiralities; ϕ representing a scalar, a typical Yukawa interaction reads: $\phi\bar{\Psi}_1\Psi_2$ + (h.c.). This term is equivalent to:

$$\phi(\bar{\Psi}_{1R}\Psi_{2L} + \bar{\Psi}_{1L}\Psi_{2R}) + \text{h.c.} \quad (2.13)$$

[**Exercise:** Establish the equivalent relation for a pseudo-scalar interaction $\phi'\bar{\Psi}_1\gamma_5\Psi_2 +$ h.c.]

Limiting ourselves to two fermions, Ψ_1 and Ψ_2, we thus suggest the picture of Fig 2.1,

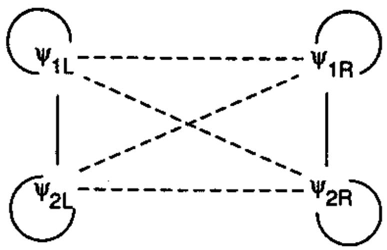


Fig 2.1

where full lines stand for allowed vector interactions, while dashed lines represent the interactions mediated by scalar bosons. This diagram further assumes the strict conservation of fermion number.

If we aim at singling out the minimum building block for gauge interactions (forgetting for the moment the problem of chiral anomalies) we thus see that a single chiral fermion (in other words a single 2-component spinor) is sufficient. For instance, with A^μ a vector boson we can write the interaction:

$$A^\mu \bar{\Psi}_{1L}\gamma_\mu \Psi_{1L}. \tag{2.14}$$

Even in this minimum scheme, Ψ_{1L} describes both the destruction of a left-handed fermion and the creation of a right-handed antifermion. The minimal content of a gauge interaction thus involves:

- one fermion of given helicity
- the antifermion of opposite helicity
- a vector boson.

Already at this level, the symmetry under charge conjugation C (which transforms a particle into its antiparticle) is not automatic, and the same is true for spatial parity (P, the simultaneous reversal of the E3 spatial coordinates). Obviously, the basic interaction introduced above can be extended on a case-by-case

basis to include more spinors, so as to respect P and C separately. Yet, at the basic level of gauge interactions, (2.14), the naturally occurring symmetry transforms a left-handed fermion into a right-handed antifermion (see for instance left-handed neutrinos and right-handed antineutrinos).

This symmetry is precisely realized by the product of operations CP, which thus appears as the basic symmetry of gauge interactions.

Even so, the present description in terms of the spinor Ψ_L still lacks symmetry, since it favours in some way particles, and it may prove useful to introduce explicitly Ψ^{CP}, which assumes a symmetric rôle. This operation is realized at the level of creation and destruction operators by defining:

$$\begin{aligned} [a_{\omega,\vec{p},\lambda}]^{CP} &= \alpha b_{\omega,-\vec{p},-\lambda} \\ [b_{\omega,\vec{p},\lambda}]^{CP} &= \alpha' a_{\omega,-\vec{p},-\lambda} \end{aligned} \tag{2.15}$$

where α and α' allow for phases in the definition of the conjugated states.

For convenience, we write down the corresponding transformation for Ψ. Equation (2.15) shows the necessity to transform from Ψ to $\bar{\Psi}$:

$$(\Psi^{CP})_i = i\gamma^0 U^c_{ij} \bar{\Psi}_j(t, -\vec{r}), \tag{2.16}$$

where U^c satisfies:

$$^t(U^c)\gamma^\mu = -\gamma^\mu U^c. \tag{2.17}$$

For further reference, we write down explicitly the part of the standard model Lagrangian describing the interaction of quarks with gauge particles:

$$\mathcal{L}_{F-J} = i\sum_j \{\bar{Q}_{Lj} D^\mu \gamma_\mu Q_{Lj} + \bar{u}_{Rj} D^\mu \gamma_\mu u_{Rj} + \bar{d}_{Rj} D^\mu \gamma_\mu d_{Rj}\}; \tag{2.18}$$

D_μ stands for the covariant derivative:

$$D_\mu \equiv \partial_\mu + igW_\mu{}^a T^a + ig'\frac{Y}{2}B_\mu, \tag{2.19}$$

with T^a the generator pertaining to the representation, i.e. $\tau^a/2$ for L fermions, but 0 for the others; Y is the weak hypercharge corresponding to the particle, and W, B represent the gauge fields respectively associated to $SU(2)_L$ and $U(1)$.

For the fun of it, we can check in a very explicit case the meaning of CP conservation. Could we suggest that the reader spends some time studying fig. 2.2, where we give a (very schematic) description of the famed Wu experiment [1] .

This experiment studies the reaction:

$$^{60}\mathrm{Co} \rightarrow \mathrm{Ni} + \mathrm{e}^- + \bar{\nu}_e \qquad (2.20)$$

in an external magnetic field. Co and Ni have spins 5 and 4 respectively. The true experiment is depicted in the lower left corner of the picture, with its mirror image just opposite, in the higher left corner. On the right-hand side of the picture, we first have an experiment, using *an imaginary apparatus that is identical to the one seen in the mirror.* The last drawing (bottom right) depicts the same imaginary experiment, with the additionnal twist that *the apparatus is now assumed to be built from antimatter.*

It is easy to interpret these diagrams (and we leave the reader to do it) with the simple assumptions that

- interactions are pure (pseudo-) vectorial
- lepton masses are negligible
- right-handed neutrinos (if such particles exist) take no part in the reaction.

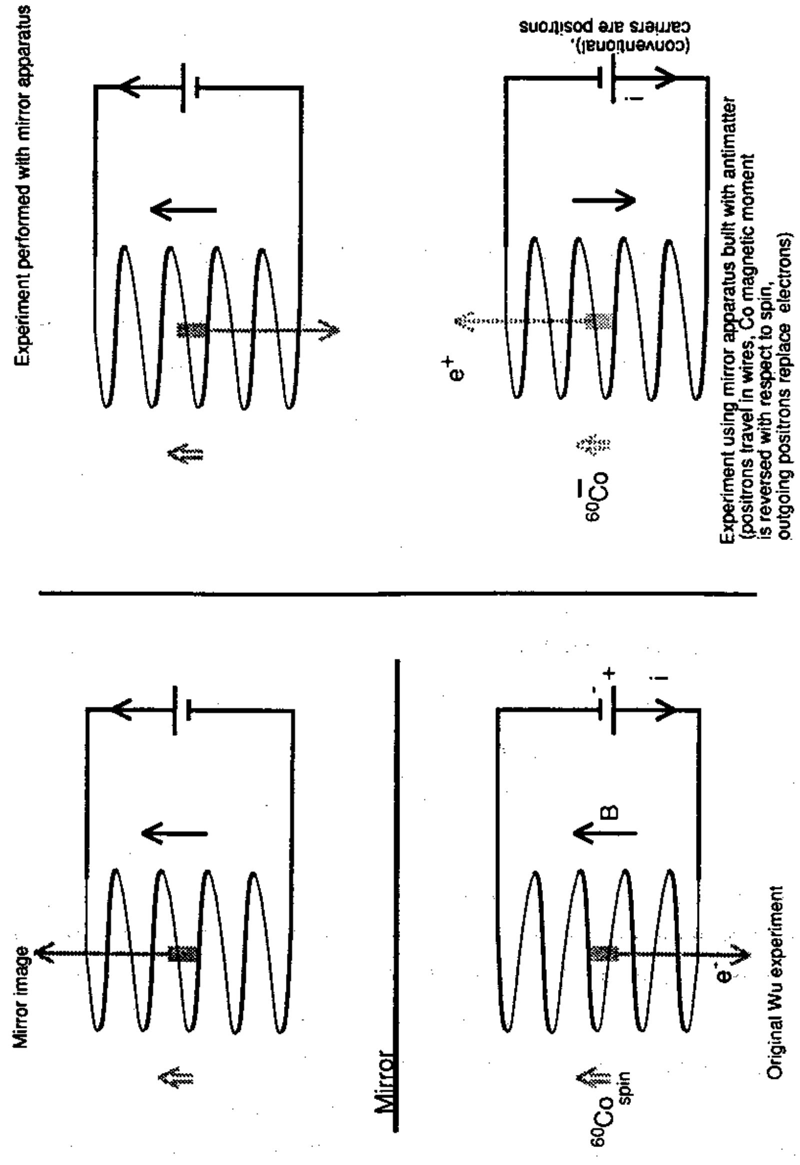


Figure 2.2

3. SCALAR FIELDS AND CP VIOLATION

At the beginning of this chapter, we limit ourselves to the "standard model", and request only invariance under $SU(2)_L \times U(1)_Y$, where Y stands for the well-known "weak hypercharge".

Within this context, and before any symmetry breaking, the distinction must be made between "right-handed quarks", which do not take part in the $SU(2)_L$ gauge interaction, and will thus be represented by the singlets u_{iR}, d_{iR} and on the other side "left-handed quarks", which sit in $SU(2)_L$ doublets:

$$Q_{iL} \equiv \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}. \tag{3.1}$$

To simplify notations, u_{iL} stands for the spinor $\Psi_{u_{iL}}$ associated to that quark. Colour indices will be omitted (all the couplings considered are colour-diagonal), while the index i stands for the "family" or "generation" to which the fermion belongs.

We insist on the fact that, up to now, these indices are to a large extent arbitrary. Nothing indeed allows to associate for instance u_{1R} to u_{1L}, since they are not related by any gauge interaction. On the contrary, d_{iL} is directly associated by gauge interaction to u_{iL}. In models with extended symmetry, e.g. LR models based on the group $SU(2)_L \times SU(2)_R \times U(1)$, a similar relation links u_{iR} and d_{iR}. As is well-known, introducing an interaction between L and R while respecting $SU(2)$ requires (if we insist on preserving the fermionic number) to introduce at least one scalar field.

In the more general case of two scalar doublets Φ_1 and Φ_2, we have:

$$\mathcal{L}_Y = \lambda^d_{ij}\bar{d}_{iR}\Phi_1 Q_{jL} + \lambda^u_{ij}\bar{u}_{iR}\Phi_2 Q_{jL} + \text{h.c.} \tag{3.2}$$

As is easily checked, the "weak hypercharges" of the doublets Φ_1 and Φ_2 must be opposite. While we know that Φ_1^* transforms equivalently to Φ_1, and has opposite hypercharge, it is more convenient to change the basis to

$$\tilde{\Phi}_1 \equiv i\sigma_2\Phi_1^*. \tag{3.3}$$

This field possesses all the desired properties, and transforms identically (instead of

equivalently) to Φ_2. Within the standard model, we can thus avoid introducing an independent Φ_2 by identifying Φ_2 to $\tilde{\Phi}_1$.

Independently of any symmetry breaking, (3.2) can break CP, to the difference of the gauge interaction (2.19). A CP transformation indeed implies a permutation of Ψ and $\bar{\Psi}$ (2.16) and thus requires a comparison of the coefficients explicitly represented in (3.2)with their hermitian conjugates. Non-real λ^u or λ^d coefficients thus lead to a CP-violating interaction Hamiltonian. Once again, this cannot happen in the gauge terms (2.18).

We want to point out however that, even if complex λ coefficients suggest some CP violation, it not necessarily observable, or even realized. We will come back to this point later.

For the reactions observed at current accelerators the $SU(2)_L \times U(1)$ symmetry breaking plays a dominant rôle (things are different in the study of the creation of the universe, and in particular of the production of the current excess of matter over antimatter, where CP violation is of crucial importance). We follow the tradition in orienting the "vacuum expectation value" (vev) of the Φ_1 field in the direction:

$$\langle\Phi_1\rangle \equiv \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}. \tag{3.4}$$

Since the electric charge is conserved, this defines the electrically neutral comp[onent of Φ_1, and simultaneously associates quarks d_{iR} and d_{jL} (through a mass term). We insist on the fact that this association, suggested by the use of the same letter for L et R partners is only justified now, in view of the special choice of orientation of the vev. For instance, we might just as well orient the vev in the direction $(v/\sqrt{2}, 0)$, which would associate u_R to d_L!

This implies no contradiction, and only highlights some abuse in the choice of our initial notations. This point settled, we observe that the mass matrices for u and d quarks are given by (assuming $\tilde{\Phi}_1 \equiv \Phi_2$, $v = v^*$):

$$\mathcal{L}_{mass} = \lambda^d_{ij}\bar{d}_{Ri}d_{Lj}v/\sqrt{2} + \lambda^u_{ij}\bar{u}_{Ri}u_{Lj}v/\sqrt{2} + \text{h.c.} \tag{3.5}$$

In the low-energy domain, and in general in disintegrations, the importance of masses usually strongly dominates that of weak interactions, and we are thus led for expediency to diagonalize (3.5). For this purpose, we use the four unitary matrices U_L, U_R, V_L, V_R, and turn to the mass eigenstates u', d':

$$
\begin{aligned}
u_{(L,R)i} &= U_{(L,R)ij} u'_{(L,R)j} \\
d_{(L,R)i} &= V_{(L,R)ij} d'_{(L,R)j}
\end{aligned}
\tag{3.6}
$$

in this basis, we get:

$$
\begin{aligned}
\frac{v}{\sqrt{2}} U_R^+ \lambda^u U_L &= \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}, \\
\frac{v}{\sqrt{2}} V_R^+ \lambda^d V_L &= \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}
\end{aligned}
\tag{3.7}
$$

where all m are assumed to be positive (the sign of a fermion mass has no meaning by itself, since it suffices to redefine, e.g. $\Psi_R \to -\Psi_R$, to flip it; only the relative sign of masses may prove important).

This operation has thus diagonalized fermion masses. As long as only ONE scalar field couples to u_R and ONE (possibly the same) to d_R, it is a trivial matter to check that the interaction of neutral scalar fields is diagonal in quark flavours (no flavour-changing neutral currents).

The same obtains for neutral vector interactions, for which the currents read:

$$
j_F^\mu = \bar{q}_i \cdot \gamma^\mu (\alpha + \beta\gamma_5) q_i, \tag{3.8}
$$

where q_i is a generic representation of $u_{L,R}$, $d_{L,R}$. The transition to eigenstates q' does not affect j^μ, thanks to the unitarity of U and V. Only charge-exchange interactions are in fact affected. For instance, the fermionic part of the left-handed charged current reads:

$$
j_F^{\mu+} = \frac{g}{\sqrt{2}} \sum_i \bar{u}_{Li} \gamma^\mu L d_{Lj} = \frac{g}{\sqrt{2}} \bar{u}'_L U_L^+ V_L \gamma^\mu L d'_L \tag{3.9}
$$

Only the product $U_L^+ V_L$ is thus observable.

[**Exercise:** Check that the same combination of U and V appears in charged scalars interations]

This product is the Kobayashi–Maskawa matrix, K:

$$
U_L^+ V_L = K. \tag{3.10}
$$

How many degrees of freedom do we expect in K?

The only real constraint on K is unitarity:

$$
KK^+ = 1. \tag{3.11}
$$

For n generations, K counts n^2 complex elements, or $2n^2$ real parameters, upon which (3.11) imposes n^2 real conditions; this leaves us with n^2 real degrees of freedom.

For convenience, we distinguish between mixing angles and phases. Comparing with a real (orthogonal) $n \times n$ matrix shows the presence of $\frac{n(n-1)}{2}$ mixing angles in general. Thus:

$$
\text{number of phases} = n^2 - n(n-1)/2 = n(n+1)/2. \tag{3.12}
$$

In principle thus, even for $n = 1$, one phase stays present, which could violate CP.

We should however observe that the "standard model" still allows for a large reparametrization invariance, this far untapped. The phase associated with a fermionic field has meaning only if we can measure it. In the absence of external sources that would be able to control individual fields, only the interactions present in the Lagrangian can be used for this purpose.

We must observe that the "standard model" is far richer in particles than in interactions suitable to constrain them. For instance, no charged interaction affects the R quarks. In a similar way, the multiplication of "families"is not associated with new gauge symmetries.

This lack of unification of the model - and in particular the presence of trivial representations of $SU(2)_L$ - lies at the origin of this large reparametrization invariance, which we now use. We have indeed every right to redefine:

$$u'_L = e^{i\alpha} \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & e^{-i(\alpha_1+\alpha_2)} \end{pmatrix} u''_L \qquad (3.13)$$

where u_L are column vectors in family space - we have limited the typography to three families, but the principle is evident. We have singled out a global phase, in order to keep unimodular matrices. For n families, we can use n phases. The same transformation - - using this time phases β_i - applies to the d_L quarks.

Quite obviously, real masses will not be kept in (3.7) unless the same transformation is applied to R quarks. As we have already observed, those matrices acting on R quarks do not appear in the final formulation [in the case of pure $SU(2)_L \times U(1)$].

The transformation (3.13) turns K into K', following the path leading from (3.10) to (3.11) and yields:

$$K' = e^{-i\alpha} \begin{pmatrix} e^{-i\alpha_1} & & \\ & e^{-i\alpha_2} & \\ & & e^{i(\alpha_1+\alpha_2)} \end{pmatrix} K \begin{pmatrix} e^{i\beta_1} & & \\ & e^{i\beta_2} & \\ & & e^{-i(\beta_1+\beta_2)} \end{pmatrix} e^{i\beta}. \qquad (3.14)$$

As appears from (3.13) the $2(n-1)$ phases present in the matrices can be used to redefine K. On the other hand, only the combination $\beta - \alpha$ (and thus not $\alpha + \beta$) appears. This transformation can thus be used to remove at most $2(n-1)+1$ phases.

Comparing with (3.12), we are left with:

$$N_{standard} = n(n+1)/2 - (2n-1) = \frac{(n-1)(n-2)}{2}, \qquad (3.15)$$

which shows clearly that at least three generations are needed to observe CP violation in the standard model, according to the mechanism suggested by Kobayashi and Maskawa.

4. A FEW WORDS ABOUT *LR* MODELS

Standard introductions to the weak interactions often insist on the crucial rôle played by the absence of right-handed neutrinos to explain parity violation. When introducing quarks, however, it is now abundantly established (e.g. in the e-deuterium scattering experiments) that, despite the presence of both u_R and d_R quarks, the L components only take part in the interaction.

The conclusion is obvious: the origin of parity violation must be found in the (broken) gauge interaction, and not simply in the fermion content. We could in fact trivially add a ν_R to the standard model, without a single observable consequence! In view of this, we could be tempted to re-establish at a higher energy scale the LR symmetry by extending the gauge group to $SU(2)_L \times SU(2)_R \times U(1)$, with all fermions now either in L or R doublets:

$$Q_{iR} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_R \quad Q_{iL} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \quad L_{i_{L,R}} = \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix}_{L,R} \qquad (4.1)$$

It is interesting to note that the new $U(1)$ acts on fermions with charges measured by the difference of their baryonic and leptonic numbers $B-L$ [which is due to the possible grand -unification by $SO(10)$]. The gauge bosons W^μ_L and W^μ_R act separately on these fermions according to the covariant derivative:

$$D^\mu = \partial^\mu + ig_L W^{\mu a}_R T^a_L + ig_R W^{\mu a}_R T^a_R + ig'\frac{(B-L)}{2}B^\mu. \qquad (4.2)$$

Except for possible differences in the coupling constants, the disparity between L and R interactions can only be explained in terms of mass differences between W_L et W_R, resulting from the broken symmetry. As a matter of fact, the standard model can be recovered as a limiting case of LR at low energy by a suitable choice of symmetry-breaking pattern, associated with supplementary scalar fields, e.g. left and right triplets.

Direct experimental limits on R-boson masses are weak, particularly if right-handed neutrinos ν_R are allowed to develop a large Majorana mass, which effectively sterilizes all R-charged leptonic currents. In any case, masses of order 500 to 600 GeV generally prove sufficient.

The set of scalar bosons responsible for the breaking of the left-over $SU(2)_L$, as well as for providing fermion masses, is also larger than in the minimal standard model, since it centres on a "bidoublet" $\Phi(1/2, 1/2, 0)$. At low energy, this resolves into the two doublets previously introduced.

$$\lambda \bar{Q}_R \Phi Q_L \equiv \lambda (\bar{u}\bar{d})_R \begin{pmatrix} \phi_2^0 & \phi_1^+ \\ \phi_2^- & \phi_1^0 \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}_L . \tag{4.3}$$

Although two vacuum expectation values are present, $\langle \phi_2^0 \rangle = v_2/\sqrt{2}$, $\langle \phi_1^0 \rangle = v_1/\sqrt{2}$, a further coupling $\lambda' \bar{Q}_R \tilde{\Phi} Q_L$ still needs to be introduced to generate a non-trivial mixing matrix. This unfortunately implies that the extra neutral scalars present in this model may mediate flavour-changing neutral interactions. The usual solution is to assume those neutral scalars to be sufficiently massive, so that their impact becomes negligible. This is possible without introducing any large couplings in the scalar Lagrangian. Indeed, only the neutral scalar corresponding to the standard model (and for which conservation of flavour is guaranteed) needs to stay light.

We do not want to dwelve any further in the details of LR models here, nor to pursue an exhaustive approach to CP violation in this framework, but wish simply to pinpoint a few of its aspects, as a comparison to standard model expectations. The reader interested in a more thorough study will find in ref. 2 the more recent estimates, and, even more importantly, the necessary references.

For the time being, we revert to counting the phases. This is actually simpler than in the standard model. We can duplicate the path leading from (3.9) to (3.10) in the R sector, thus building a second observable mixing matrix (we thus refer now to K_L and K_R). Each of these matrices contains $n(n-1)/2$ mixing angles and $n(n+1)/2$ phases.

As an arbitrary convention, we can follow the prescription leading to (3.13) and eliminate as many phases as possible from K_L. Now, however, a simultaneous rotation of R quark phases, necessary to keep (3.7) will directly affect K_R. Even for $n < 3$ it thus proves in general impossible to eliminate simultaneously all phases from both K_L and K_R.

Assuming that we have completed the above procedure, we now count $(n-1)(n-2)/2$ phases in K_L while being left with $n(n+1)/2$ phases in K_R. If no mixing

between W_L and W_R were present, this number could still be reduced by one unit. Let us write indeed $K_R = e^{i\alpha} K'_R$. In all quark interactions or disintegrations where W_R is emitted and reabsorbed, the phase α automatically cancels out. Mixing between W_L and W_R however does occur, either through scalar interactions or through quark loops (as usual the crosses in this diagram stand for chiral transitions associated with quark masses).

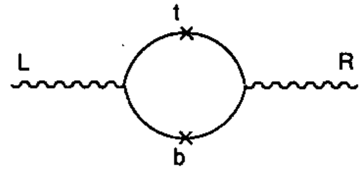


Figure 4.1

Physical gauge bosons thus appear as mixed states W_1 and W_2. In practice the mixing is weak enough ($< 0.5\%$) to be handled as a mere perturbation.

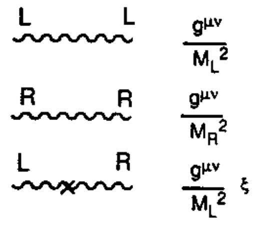


Figure 4.2

At low energy, we must thus take into account the W_L, W_R propagators, and their mixing. Neglecting the momentum transfer in front of the mass terms, this yields respectively $\frac{g^{\mu\nu}}{M_L^2}$, $\frac{g^{\mu\nu}}{M_R^2}$ and $\frac{g^{\mu\nu}}{M_L^2} \cdot \xi$ for the W_L, W_R, and mixed propagators.

The presence of LR mixing thus allows for the virtual emission of a W_R, its transformation into a W_L is later reabsorbed. The corresponding amplitude is thus proportional to $K_L^+ K_R$, which makes the overall phase in K_R observable! As a result, CP violation in a LR model may occur even with only one generation! (independently of any violation due to strong interactions and anomalies).

Among the possible consequences we find for instance the possibility of a large neutron belectric dipole moment. We will return to this comparison of standard and LR models of CP violation, but we now need some more phenomenological input.

5.CP VERSUS TCP

In this section, we return to a more phenomenological discussion of the nature of CP violation. One of the most spectacular examples of CP violation would consist in showing an explicit difference in the behaviour of particle and antiparticle, as was suggested in the introduction. The most basic properties of a particle are its mass and lifetime. Differences in mass or lifetime would certainly constitute a form of CP violation, since no P effect appears in the definition of mass, and since the lifetime calculation results from a sum over all possible disintegration modes, which ensures that this notion is P–symmetric too. Unfortunately, such spectacular effects are forbidden in the framework of local Lagrangian theories, as a result of the CPT theorem.

Considering for example the evolution equation for a K^0, the matrix elements of the evolution operator S (or of the Hamiltonian), $\langle K^0|S|K^0\rangle$ and $\langle \bar{K}^0|S|\bar{K}^0\rangle$, are directly related by TCP, which implies the equality of masses and lifetimes for particle and antiparticle. This equality however only applies to the total lifetime of the particle : different partial decay rates are allowed, as long as the total balance is respected.

This paragraph will address the K^0 system, temporarily neglecting for pedagogical reasons a dominant effect, K^0-$\bar{K}^0$ mixing. Although apparently academic, this study is important, for it sheds some light on the meaning of a notion that is often hard to grasp, the ϵ' parameter describing explicit CP violation in K's, and opens the way to the study of similar processes in heavy–quark systems,in particular for B's.

CPT thus requires :

$$\tau^{-1}(K^0) = \Gamma(K^0 \to x)+\Gamma(K^0 \to y)+\ldots = \Gamma(\bar{K}^0 \to \bar{x})+\Gamma(\bar{K}^0 \to \bar{y})+\ldots = \tau^{-1}(\bar{K}^0) \tag{5.1}$$

but does not imply the equality of $\Gamma(K^0 \to x)$ and $\Gamma(\bar{K}^0 \to \bar{x})$. Should a difference should appear, for instance :

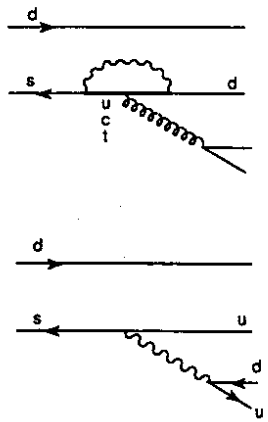
$$\begin{aligned} \Gamma(K^0 \to 2\pi^0) &= a + x \\ \Gamma(\bar{K}^0 \to 2\pi^0) &= a - x \,. \end{aligned} \tag{5.2}$$

one or more other decay channels will have to balance it in (5.1). For consistency,

these other channels need to appear explicitly when computing (5.2).

To show this we limit ourselves, for simplicity, to the (dominant) 2π decays (mixing effects will be neglected, as already announced). Various bases can be used to describe the 2π decays. Two choices appear naturally : one is proper to strong interactions, with states described in terms of isospin; in the other, more appropriate for a macroscopic description (in this we include the interaction with dectectors), the leading rôle is taken by the electric charge. Those bases are related by :

$$\begin{aligned} |2\pi, I=2\rangle &= \frac{1}{\sqrt{6}}(\pi^+\pi^- + \pi^-\pi^+ + 2\pi^0\pi^0) \\ |2\pi, I=0\rangle &= \frac{1}{\sqrt{3}}(\pi^+\pi^- + \pi^-\pi^+ - \pi^0\pi^0) \,. \end{aligned} \tag{5.3}$$



Figures 5.1a ,5.1b

Working first in the isospin bases, we notice that the various diagrams associated with the decays (fig. 5.1) do not contribute equally to the various isospin channels (for instance, fig. 5.1a only contributes to $I=0$, while 5.1b feeds the two amplitudes). The three quark families take part in 5.1a so that a CP-violating phase can in general appear while this cannot be the case for 5.1b. Different "weak" phases thus affect the amplitudes associated with the two channels: they will be represented by ξ_0 and ξ_2. If the amplitude describing $K^0 \to 2\pi$, $I=0$ receives a "weak" phase ξ_0, the conjugated amplitude $\bar{K}^0 \to 2\pi$ will instead bear the phase $-\xi^0$, as results clearly from (3.9) and its Hermitian conjugate.

We cannot however neglect the presence of strong interactions (fig. 5.2). They appear diagonal in the isospin bases, and act to provide phases δ_0 and δ_2 respectively. These phases are only related to the isospin state of the pions, irrespective of their origin! In particular, their do not vary when going from K^0 to $\bar{K}^0$.

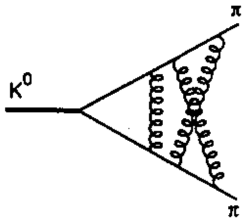


Figure 5.2

We thus write:

$$
\begin{aligned}
A(K^0 \to 2\pi, I=0) &\equiv A_0 = a_0 e^{i\xi_0} e^{i\delta_0} \\
A(K^0 \to 2\pi, I=2) &\equiv A_2 = a_2 e^{i\xi_2} e^{i\delta_2} \\
A(\bar{K}^0 \to 2\pi, I=0) &\equiv \bar{A}_0 = a_0 e^{-i\xi_0} e^{i\delta_0} \\
A(\bar{K}^0 \to 2\pi, I=2) &\equiv \bar{A}_2 = a_2 e^{-i\xi_2} e^{i\delta_2}
\end{aligned}
\qquad (5.4)
$$

If we first study the differences between the partial widths $\Gamma(K^0 \to 2\pi, I=0)$ and $\Gamma(\bar{K}^0 \to 2\pi, I=0)$, we notice immediately that (5.4) yields a null result since both are found to be proportional to $|a_0|^2$.

This result is expected, since

- no other channel suitable for compensating a difference in lifetime appears explicitly in the calculation,
- the phase ξ_0 could in any case have been eliminated by a redefinition of the quark fields (at the cost of redefining ξ_2 accordingly).

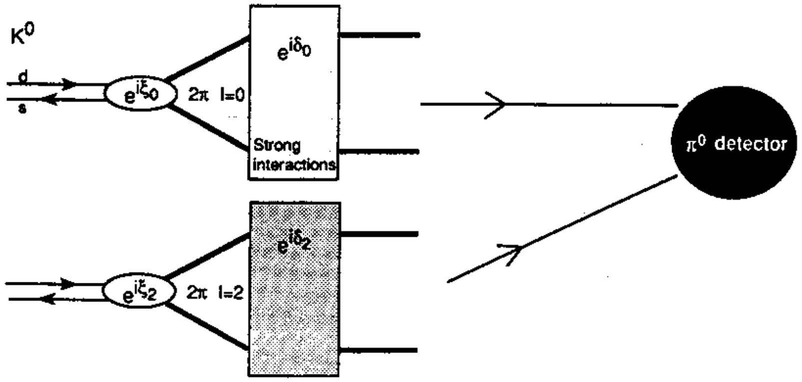


Figure 5.3

A non-vanishing effect requires some interference between the various terms appearing in (5.4). Such an interference is introduced by the macroscopic apparatus, since pions, once separated, feel mainly electromagnetic forces. In particular, the detectors obviously do not select isospin eigenstates, but charged particles (cf. fig. 5.3). For the neutral mode, we thus get :

$$
A(K^0 \to 2\pi^0) = \frac{-1}{\sqrt{3}} a_0 e^{i\xi_0} e^{i\delta_0} + \sqrt{\frac{2}{3}} a_2 e^{i\xi_2} e^{i\delta_2} \qquad (5.5)
$$

which yields, up to common kinematic factors :

$$
\begin{aligned}
\Gamma(K^0 \to 2\pi^0) &\sim \frac{1}{3}a_0^2 + \frac{2}{3}a_2^2 - \frac{2\sqrt{2}}{3} a_0 a_2 \cos(\xi_0 - \xi_2 + \delta_0 - \delta_2) \\
\Gamma(\bar{K}^0 \to 2\pi^0) &\sim \frac{1}{3}a_0^2 + \frac{2}{3}a_2^2 - \frac{2\sqrt{2}}{3} a_0 a_2 \cos(-\xi_0 + \xi_2 + \delta_0 - \delta_2)
\end{aligned}
\qquad (5.6)
$$

Equations (5.6) thus show a difference between the branching ratios for $K^0 \to 2\pi^0$ and $\bar{K}^0 \to 2\pi^0$. The interference between modes a_0 and a_2 is obvious, and identifies the compensating channel, namely $\pi^+\pi^-$ (check!). Furthermore, the simultaneous need for "weak" phases ξ and for interferences mediated here by the strong interactions δ clearly appears in (5.6).

The difference between partial widths indeed vanishes when $\delta_0 = \delta_2$. This example also shows that a precise knowledge of strong interaction phase shifts is needed to extract the weak phase difference $(\xi_0 - \xi_2)$ experimentally. For the K system, this requirement is relatively easy to meet, in particular since so few channels are involved (unitarity arguments can also be invoked in this special case, see for instance ref. 3). This determination would however prove considerably harder in heavier systems, and sensitive tests of the standard model would only be possible for signals that are independent of these strong phases.

Before closing this chapter, a few more remarks are in order :

- The difference in branching ratios between $K^0 \to 2\pi^0$ and $\bar{K}^0 \to 2\pi^0$ is related in the above calculation to $(\xi_2 - \xi_0)$. This combination is precisely the one appearing in the definition of the CP-violating parameter ϵ' (see below). Fortunately, more refined methods can be used to measure ϵ'; the above interpretation of this parameter is however useful and justifies the frequently used expression "explicit CP violation".
- The reader might be surprised to see only gauge interactions in fig. 5.1, while we insisted at length on the fact that scalar interactions were responsible for CP violation. In fact, the low-energy situation forces us to diagonalize the quark masses, that is to include effectively scalar interactions both in external states and in fermion propagators. In this basis, CP violation is (artificially) transferred to the non-diagonal character of the gauge interaction.
- In the previous section, we have taken much freedom in redefining phases associated with fermion fields, with the claim that (in the absence of anomalies) only the phases left over after such reparametrization could contribute to CP violation. It might prove useful for the reader to check this affirmation in the case upon consideration, for instance by redefining arbitrarily the s-quark field: $s' = e^{i\alpha}s$, and checking the impact on A_0, A_2.

6.THE K SYSTEM

The only current evidence for CP violation is found in the K_0 system. Two peculiarities make this system unique, and made this observation possible in the first place. The first issue is mixing. Weak interactions-independently of any CP effect-mix (at the second order in G_F) the strong interaction eigenstates K_0 and $\bar{K}_0$.

Ideally, if CP were conserved, this would lead to the eigenstates :

$$\begin{aligned} |K_1^0\rangle &\equiv \frac{1}{\sqrt{2}}(|K_0\rangle + |\bar{K}_0\rangle) \qquad CP = 1 \\ |K_2^0\rangle &\equiv \frac{1}{\sqrt{2}}(|K_0\rangle - |\bar{K}_0\rangle) \qquad CP = -1 \,. \end{aligned} \tag{6.1}$$

Since the 2π final state has $CP = +1$ (this is specially easy to check in the special case $\pi^0\pi^0$), CP conservation would imply

$$K_2^0 \not\to 2\pi \,. \tag{6.2}$$

Here kinematics help – and this is the second important peculiarity – since the next hadronic mode* into 3π, accessible to K_2^0, is strongly suppressed by phase space. A huge lifetime difference between the two final states follows.

We thus expect that in a K^0 beam, the K_1^0 component quickly decays the (2π) decay mode disappears, so that with only the K_2^0 component left. A 2π decay of the long-lived eigenstate (which we now call K_L^0, in opposition to the shorter-lived K_S^0) is however observed. This effect is traditionally expressed in terms of the ratio of amplitudes :

$$\frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \simeq \epsilon \tag{6.3}$$

The source of CP violation in (6.3) could be located both in the disintegration mode of K^0 and $\bar{K}^0$ (as seen in the previous chapter), in the diagonalization procedure leading to the physical states K_L^0 and K_S^0. We limit ourselves, as far as the time

* $K_1^0 \to 3\pi$ is not completely forbidden by CP considerations, but the final state with vanishing angular momentum between the 3π is excluded.

evolution of K^0's is concerned to the three channels ($\bar{K}^0, 2\pi,\ I = 0, I = 2$), and must now consider the three phases associated with the amplitudes

$$A(K^0 \to \bar{K}^0) = M_{12}$$
$$A(K^0 \to 2\pi, I = 0) = a_0 \cdot e^{i\xi_0} \cdot e^{i\delta_0} \tag{6.4}$$
$$A(K^0 \to 2\pi, I = 2) = a_2 \cdot e^{i\xi_2} \cdot e^{i\delta_2}\,.$$

We further stress that the matrix M_{ij}, which describes the evolution of the limited K^0–$\bar{K}^0$ system according to:

$$i\partial_t \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix} = (M_{ij}) \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix} \tag{6.5}$$

cannot be considered hermitian: the $K^0 - \bar{K}^0$ subsystem is indeed not closed by itself, and a loss of probability must be allowed to take the decays into account (in particular the 2π disintegrations). We must thus write :

$$M_{ij} = m_{ij} - i\frac{\Gamma_{ij}}{2}\,, \tag{6.6}$$

with both matrices now Hermitian. Discussing CP violation within the limited K^0–$\bar{K}^0$ framework is meaningless. CP violation is indeed actually observed through the decays.

This trivial remark finds its counterpart in the definition of phases. There is indeed no meaning associated with the imaginary part of m_{ij}! A mere redefinition of quark fields $s \to e^{i\alpha}s$ and $\bar{s} \to e^{-i\alpha}\bar{s}$ suffices to cancel such an imaginary part. As we expect, on physical grounds, only the relative phase between m_{ij} and the decay amplitudes is significant. (See the exercise at the end of the previous chapter).

The usual choice consists in setting arbitrarily $\xi_0 = 0$, a choice justified since the $I = 0$ mode is largely dominant (a factor 20 for the amplitudes) over the mode $I = 2$. In practice, this allows us to neglect the phase assigned to Γ_{12}. For more details we refer the reader to the very clear presentation of Bell and Steinberger[3].

With this choice of phases, and after a few simplifications, we obtain :

$$\frac{A(K_L^0 \to \pi^+\pi^-)}{A(K_S^0 \to \pi^+\pi^-)} = \epsilon + \epsilon' \tag{6.7}$$

$$\frac{A(K_L^0 \to \pi^0\pi^0)}{A(K_S^0 \to \pi^0\pi^0)} = \epsilon - 2\epsilon' \tag{6.8}$$
$$\epsilon = \frac{e^{i\pi/4}\mathrm{Im}(m_{12})}{2\sqrt{2}\mathrm{Re}(m_{12})} \tag{6.9}$$
$$\epsilon' = \frac{1}{\sqrt{2}} e^{i(\Gamma/2+\delta_2-\delta_0)} \frac{\mathrm{Im}(A_2 e^{-i\delta_2})}{A_0 e^{-i\delta_0}} \tag{6.10}$$

[we had to introduce δ_0 and δ_2 in (6.10) to remain consistent with our definition (5.4)].

The eigenstates K_L and K_S now read:

$$K_L = \frac{1}{\sqrt{1+|\epsilon|^2}}(K_2^0 + \epsilon K_1^0)$$
$$K_S = \frac{1}{\sqrt{1+|\epsilon|^2}}(K_1^0 + \epsilon K_2^0)\,. \tag{6.11}$$

The factor $e^{i\pi/4}$ appearing in (6.9) results from an approximation of the ratio $\Delta\Gamma/\Delta m$. The fact that $\Delta\Gamma$ and Δm are comparable in the K system is responsible for the value of the phase of the parameter ϵ. The relative phase of ϵ' and ϵ is close to zero.

From (6.11), (6.7), and (6.8) we see that if the part of CP violation described by ϵ can be blamed on the K^0–$\bar{K}^0$ mixing, ϵ' cannot, since this parameter affects the channels $\pi^0\pi^0$ and $\pi^+\pi^-$ differently. In practice, most of the observed CP violation is due to the phase difference between m_{ij} and the dominant disintegration amplitude A_0. As previously announced, the parameter ϵ' measures the phase difference $\xi_2 - \xi_0$, and is thus directly related to the "explicit CP violation" described in the previous section.

The smallness of the ratio ϵ'/ϵ is suggestive of the "superweak" models. Even though this appellation may have evolved in time, we will use this term for models where CP violation in the K_0 system can be reduced entirely to the $\Delta S = 2$ sector (as is the case for ϵ).

The standard model predictions for ϵ'/ϵ have considerably varied over the years, partly as a result of the uncertainty of some experimental parameters (m_t, mixing angles), but more importantly because of the difficulty to evaluate the matrix elements of the relevant operators in an energy domain where strong interactions are dominant. The current values agree with experiment (ref. 4).

We should stress two points related to CP violation in the K system :

- CP violation in this low-energy domain is extremely small, even more so than suggested by the experimental value $|\epsilon| = 2.2 \times 10^{-3}$;
- the small experimental value for $\epsilon'/\epsilon = (2.2 \pm 1.1) \times 10^{-3}$ (given by the Particle Data Group) showing that ϵ'/ϵ is at most of order 10^{-3}, might appear as a strong argument in favour of "superweak" models; this conclusion might however be misleading.

We start with the second point (the first question will be addressed in the next chapter). Going back to the definition of ϵ' (6.10), or to our first approach to CP violation, we observe that such effects require the inteference of a_0 with a_2. For reasons largely foreign to CP $|a_0| \simeq 20|a_2|$, and this disparity of the amplitudes effectively imposes a suppression a_2/a_0 an any explicit CP violation in this system. The "superweak" or otherwise nature of CP violation should thus rather be discussed by comparing (up to $\sqrt{2}$ factors) $|\epsilon'/\epsilon|$ to the naï ve value $|a_2/a_0| \sim 5 \times 10^{-2}$. For the present experimental value, the suppression barely exceeds one order of magnitude.

7. ϵ, ϵ' AND THE LR MODEL

We have mentioned at the end of the previous section that the extent of CP violation was in a way overestimated by the parameter ϵ. The reason for this is well-known : if we follow the standard convention of setting ξ_0 to zero, the calculation of ϵ reduces to eq. (6.9) where m_{12} is given by $\langle \bar{K}_0 | S | K_0 \rangle$. The numerator of this expression then measures the observable CP violation. The convention however scales this quantity by the mass difference between the K_L^0 and K_S^0 masses, a notoriously minute quantity.

The absence of Neutral Flavour Changing current indeed forbids this mass difference below the 4th order in g. The Glashow–Iliopoulos–Maiani suppression further reduces the importance of the "box" diagram. It might be useful, in order to understand the unique nature of LR models, to review briefly this GIM mechanism; the following discussion refers to the graph in fig. 7.1.

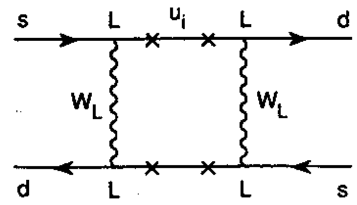


Figure 7.1

The evaluation of this diagram in principle requires the integration

$$\sum_{i,j} \int d^4k \left(\gamma_\mu K L \frac{\not{k} + m_{ui}}{k^2 - m_{ui}^2} \gamma_\nu K^+ L \right)_{21} \left(\gamma_\nu K L \frac{\not{k} + m_{uj}}{k^2 - m_{uj}^2} \gamma_\mu K^+ L \right)_{21} \left(\frac{1}{k^2 - M_L^2} \right)^2 , \tag{7.1}$$

where L stands for the left-handed projector, and where external momenta have been neglected; M_L^2 is the gauge boson mass (we will not consider here radiative corrections). At first sight, the integral seems to diverge if k^2 is neglected in front of M_L^2.

For large $|k|$, however, the dominant term vanishes. For $|k| \gg m$ indeed,

$$\gamma^\mu K L \frac{\not{k}}{k^2} \gamma^\nu K^+ L \sim (K K^+)_{21} = 0 \tag{7.2}$$

since $KK^+ = 1$. The purely L couplings further forbid contributions linear in m on any fermion line (counting the L projector makes this obvious). Since the same reasoning applies to both fermion propagators, the integral turns out to be convergent even without using the momenta in the gauge-bosons propagators. This reasoning is represented on the graph by mentioning explicitly (using crosses) the mass factors necessary for each propagator. The resulting matrix element is then:

$$\langle \bar{K}_0 | \bar{s} \gamma^\mu \frac{1 - \gamma_5}{2} d \, \bar{s} \gamma^\mu \frac{1 - \gamma_5}{2} d | K_0 \rangle = 1/3 \, B \, f_K^2 \, m_K . \tag{7.3}$$

This matrix element is in turn chirally suppressed, a fact shown here by the factor f_K. The fudge factor B accounts for the ratio between the exact value of the matrix element and an evaluation allowing only for the vacuum as an intermediate state. The exact value of B is uncertain, extreme numbers ranging from 1/2 to 3 have been suggested.

Let us further mention that in the standard model, as already stated the three families must contribute to allow any CP violation, and that in particular the tiny mixing angles with the 3rd generation must enter as factors. After radiative corrections,

$$|\epsilon|_{SM} \simeq 1.34 s_2 s_3 \sin\delta(1 + 860\ S\left(\frac{m_t^2}{M_L^2}\right) s_2\ \mathrm{Re}\ K_{ts})$$
$$S(x) \simeq x\left(\frac{1}{4} + \frac{9}{4}\frac{1}{1-x} - \frac{3}{2}\frac{1}{(1-x)^2}\right) - \frac{3}{2}\left(\frac{x}{1-x}\right)^3 \log x \ . \tag{7.4}$$

A discussion of radiative corrections leading to this result can be found in the literature (e.g. ref. 4).

The situation is quite different in LR models. We temporarily neglect the mixing ξ between L and R, and observe that the LR counterpart to the box diagram is far less suppressed.

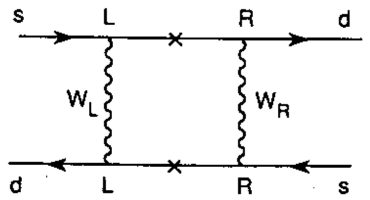


Figure 7.2

Even assuming $K_L \equiv K_R$, the alternance of L and R projectors on the leptonic lines allows for a linear contribution of quark masses. Neglecting k^2 in W propagators would lead to a divergence, and this results in a $\ln \frac{m_{quark}}{M_L}$ dependence of the result. The chiral structure of the resulting matrix element is also different, and the previous suppression is mitigated by a factor $\frac{m_K}{m_s}$. The consequence is a considerable enhancement of the K^0–$\bar{K}^0$ transition in the LR model, as was noticed by G. Beall et al.[5]. To make this explicit, we find it convenient to write

$$\langle K^0 | S | \bar{K}^0 \rangle_{LL} = A \cdot \frac{1}{M_L^2} \cdot \frac{1}{M_L^2} \ . \tag{7.5}$$

and in a similar way

$$\langle K^0 | S | \bar{K}^0 \rangle_{LR} = A \cdot \frac{1}{M_L^2} \cdot \frac{1}{M_R^2} X \tag{7.6}$$

where X is the global enhancement factor resulting from the above–mentioned effects. Taking into account radiative corrections, we get $X = 230$ (an update of those values and useful references can be found in ref. 2). The first implication is a very strong limit on the value of M_R. The standard model calculation of the mass difference between K_0 and $\bar{K}_0$ having the correct order of magnitude, the LR contribution should not upset this result. Taking into account the uncertainties in the evaluation (in particular the B factor), a LR contribution at most equal to the usual one is generally admitted. Assuming $K_L = K_R$, this leads to

$$M_R \geq 1.6 \text{ TeV} \ . \tag{7.7}$$

More importantly for our purpose, the enhancement (7.6) is exactly what is needed to grant a "superweak" CP violation. We will show indeed that such an enhancement is absent from $\Delta S = 1$ transitions. Furthermore, we have already shown that LR models involve more phases than the standard case, which allows for CP violation in the K system even without any contribution from the 3rd family of quarks.

The reason why ϵ' is not enhanced in a way similar to ϵ in LR models is very simple. The processes contributing to ϵ', that is to the phases ξ_0 and ξ_2 of the $\Delta S = 1$ amplitudes contain only one W boson (contributions of higher order in G_F are obviously negligible). Radiative corrections due to strong interactions are blind to the L or R chirality of W couplings, as long as LR mixing is neglected.

We can easily estimate the order of magnitude of expected contributions. Let us assume indeed that CP violation is dominated by the phases associated with W_R exchange. This would in particular be the case if mixing angles to the 3rd family were negligible. This yields :

$$|\epsilon| = \frac{1}{2\sqrt{2}} \cdot \frac{\mathrm{Im}(m_{12})}{\mathrm{Re}(m_{12})} \simeq \frac{A M_L^{-2} M_R^{-2} X}{A M_L^{-2} M_L^{-2}} \sin\phi \ , \tag{7.8}$$

where we assumed M_R to be large enough for the essential contribution to $\mathrm{Re}(m_{12})$ to be due to the L sector alone, and where ϕ is some combination of the phases involved.

For the $I = 0$ and $I = 2$ amplitudes, we obtain typically :

$$\xi_{0,2} \simeq \frac{M_R^{-2} \sin \phi'_{0,2}}{M_L^{-2}} \tag{7.9}$$

with ϕ'_0 and ϕ'_2 suitable phase combinations. Taking into account $|a_0/a_2| \sim 20$, we get :

$$\left|\frac{\epsilon'}{\epsilon}\right| \sim \frac{2}{20} \frac{\sin \phi'}{\sin \phi} \frac{1}{X} . \tag{7.10}$$

If we further assume that ϕ' and ϕ are of the same order of magnitude, we obtain a typical value for (ϵ'/ϵ) of order 3×10^{-4}. This is quite obviously a mere order of magnitude, reached at the cost of rather drastic approximations.

The large number of parameters of LR models unfortunately makes it a hard task to improve this limit in all generality. A special case is however worthy of study, and depicts a situation where both P and CP violation occur spontaneously, i.e. one can choose a basis where all Yukawa couplings (see section 3) are simultaneously real, and where phases only occur between the vacuum expectation values v and v' of the scalar doublets.

Such models were studied by Chang[6], and later in a more detailed way by Ecker and Grimus[7]. In this special case, all phases originate from the ratio v/v'^*, and it turns out that the Yukawa couplings can be reconstructed in full from this ratio and the experimentally known quarks masses and mixings. This task was undertaken in a perturbative way by Chang, and completed in all generality in ref. 2. This work allows the prediction of ϵ'/ϵ as a function of v/v'^*.

Figure 7.3 provides an exemple of such predictions, as a function of the phase of v/v'^* and of the W_R mass.

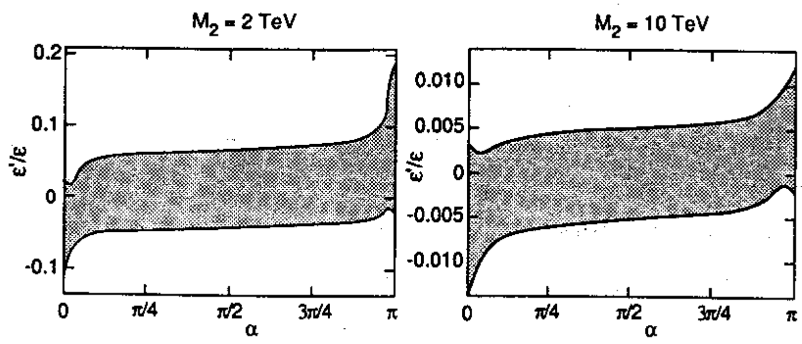


Figure 7.3

Such predictions spread over a large ribbon, which is due on the one hand to experimental uncertainties, and on the other to the existence of several branches in the inversion procedure which determines the Yukawa couplings as a function of the quarks masses and mixings.

8. *LR* AND STANDARD MODELS: OTHER CHANNELS

In view of the considerable uncertainties remaining in the evaluation of the parameters ϵ and ϵ' (in particular in the standard model), we can hardly expect to find there a totally convincing verification of a model (the small current value for the ratio can however be used to exclude some approaches). The study of other, more specific channels is thus necessary to discriminate between various theories.

B decays are an obvious direction in which to turn, to the extent that specific channels, insensitive to calculation problems, can be isolated. We will briefly return to this question at the end of the current section. A different approach consists in the investigation of various processes, where some of the possible models (*LR*, suspersymmetry, extra scalars, ...) predict or allow for important contributions while others can only yield negligible effects. We will briefly consider here some of these observables.

8.1. Neutron Electric Dipole Moment (EDM)

Such an EDM was already mentioned as an example of CP or T violation in our introduction. Let us review why such an EDM constitutes a violation of CP. By itself, an electric dipole moment is perfectly allowed by electromagnetism, and a familiar example is that of water molecules. For an elementary particle however, the only favoured direction would be that of the spin (in agreement with its description as a Dirac spinor) and an EDM could thus only align on the spin. Under T, the EDM does not flip sign, while the spin does; the projection of the EDM on the spin thus cannot be invariant under T, or, assuming CPT to be conserved, under CP.

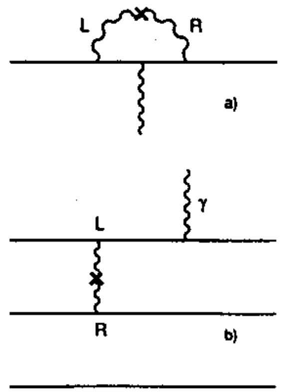
An EDM is represented in the effective Lagrangian by:

$$\mathcal{L}^{eff}_{EDM} = i\alpha\epsilon\bar{\psi}\sigma^{\mu\nu}\gamma_5 F_{\mu\nu}\psi. \qquad (8.1)$$

where $F_{\mu\nu}$ is the electromagnetic field tensor.

As a result, the EDM is necessarily associated with a *LR* transition for fermions (see section 2). It comes thus as no surprise that relatively large EDM's arise in theories where CP violation is due to scalar or both to *L* and *R* interactions.

For the standard model, however, we have seen that the mass-matrix diagonalization could be used to confine CP violation at low energy in the charged gauge bosons sector (and these have pure *L* coupling). This and the fact that contributions from the three fermion families are needed result in extremely low estimations for the quark or neutron EDM in the standard model.



Figures 8.1 a et b

In *LR* models on the contrary, this type of *CP* violation automatically arises through *LR* mixing of the gauge bosons. Contributions arise already for two families (fig. 8.1a) or even for one family (fig. 8.1b)! As a matter of fact, the predicted values are close to the current experimental limits.

$$\begin{aligned} d_n &= (-1.4 \pm 0.6) \times 10^{-25}\text{e.cm (Leningrad, 1986)} \\ &= (-0.6 \pm 0.6) \times 10^{-25}\text{e.cm (Grenoble, 1988)} \end{aligned} \qquad (8.2)$$

In the particular case of spontaneous P and CP violation, when we constrain the couplings to yield the experimental value of ϵ, ϵ' is predicted as a function of the phase of $v \times v'$. The EDM of the neutron, d_n, is also predicted as a function of that phase, which allows us to produce exclusion plots in the $\epsilon'/\epsilon : d_n$ plane.

Figure 8.2 provides some examples of these diagrams.

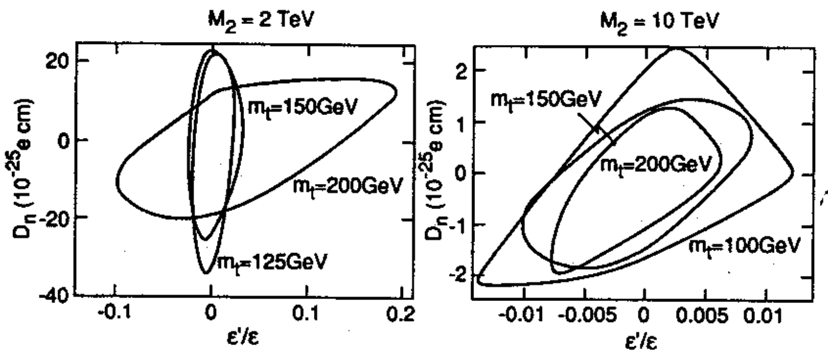


Figure 8.2

Experimental uncertainties combine with the large number of different branches appearing (see *supra*) in the inversion process to determine the Yukawa couplings from observed masses, and lead to rather wide areas in these plots. In principle however, each of the discrete solution corresponds to a simple line, and the areas depicted here are the envelopes of the actual solutions. For more details, see ref 2.

8.2. "T-ODD" OPERATORS

We use deliberately the expression "T-odd" instead of "T-violating" for these operators, since they are not necessarily good signals of T violation. If we consider for exemple the decay $K \rightarrow \mu, \bar{\nu}, \pi$, the μ polarisation with respect to the decay plane is defined by:

$$\kappa = \vec{\sigma}_{\mu}.(\vec{p}_{\mu} \times \vec{p}_{\pi}). \tag{8.3}$$

Under a pure reversal of the kinematic variable t, the three quantities appearing in (8.3), and thus κ, flip sign. This however does not depict the full effect of T reversal as the latter would also require an interchange of initial and final states, [i.e. the process $\pi + \mu + \bar{\nu} \rightarrow K$ (!)]. Thus κ is not really a pure signal of T violation, and, as a matter of fact, radiative corrections to the final state could contribute a non-zero value (in particular for $\bar{K}^0 \rightarrow \pi^+\mu^-\nu^-$).

If the importance of radiative corrections is safely examined, and found to be small (or if a comparison with the charge-conjugate channel is used to exclude such

non-violating contributions), "T-odd"operators can yield useful information. To the difference of models with extra scalars[8], no contribution to (8.3)[9] is expected in LR models for purely kinematic reasons. On the contrary, the decay $K_L \rightarrow \pi^+\pi^-\mu^+\nu$ deserves an experimental re-examination[10].

This situation of "T-odd" observables is comparable with the case of forward–backward asymmetries in the scattering $e^+e^- \rightarrow \mu^+\mu^-$. Here also, asymmetries in $(\langle \vec{p}_{e^-} \cdot \vec{p}_{\mu^-} \rangle)$ are NOT by themselves a signal of P violation, and can occur through higher-order corrections in pure quantum electrodynamics. Yet, in the existing interpretative framework, their large values can only be understood in terms of axial couplings for the Z boson, and these asymmetries are thus *de facto* an indication of Pviolation in neutral currents.

8.3. CP VIOLATION IN THE LEPTONIC SECTOR

We only mention this class of decays to stress that they are the obvious sector where NO effects are expected from the standard model. Unfortunately, in a minimal LR model CP violation effects in this sector are closely related to the scale of mass of light neutrinos. As a mere example, it is however easy to construct simple extensions, involving one extra two-component "singlet" fermion per generation, and allowing at the same time for massless neutrinos and large CP-violating effects. Such effects could in particular induce a sizeable electron EDM, or polarization in $K \rightarrow \mu^+\mu^-$. Such constructions have essentially the merit to show that CP-violation effects in the leptonic sector are in no way excluded by field theory, and that experimental studies should thus not be discouraged[11].

8.4. LR MODELS AND B DECAYS

We have not to date produced diagrams similar to those in fig. (8.2) in the case of B decays. One of the problems in such an enterprise would be the much stronger sensitivity to the yet badly determined part of the mixing matrix. An analytical study by Ecker and Grimus[12] is exemplifies of the situation, although it uses some older, and thus low, values for the top mass. The tendency is however clear, and can be understood from the reasoning we made to explain the enhancement of CP violation in the $\Delta S = 2$ channel in LR models (cf. section 7). We have shown indeed that two of the essential elements in that enhancement were the presence of logarithms $\ln \frac{m_c}{M_W}$, and the lack of chiral suppression, resulting in a factor m_K/m_s. Both of these effects loose their importance if m_t replaces m_c, while m_B and m_b fill the roles of m_K and m_s.

In other words, the enhancement of CP observed for LR models in the K system tends to vanish for the B system, while on the contrary, CP violation effects due to the standard model tend to be relatively larger. In general terms, we thus expect weaker effects in the LR framework. In the case of B_S, however, because the standard model predictions are small, Ecker and Grimus get comparatively larger effects.

These general considerations could however be invalidated if some exceptional phase combinations occur (in particular in more general LR models where spontaneous P and CP violation is not assumed).

8.5. LR MODELS AND THE BARYON ASYMMETRY OF THE UNIVERSE

While space does not allow us to develop this point here, we do not want to end this brief overview of the subject without mentioning the important question of the origin of the baryon number of the Universe.

Assuming that the initial state was symmetric under C, Sakharov has shown that three conditions were needed to generate the presently observed excess of matter over antimatter, namely, CP violation, an out-of-thermodynamical-equilibrium phase, baryon-number violation. Even the standard model has all the ingredients to satisfy these conditions, in particular if unstable solutions such as sphalerons are taken into account. To our knowledge, however, at the time of writing, the amount of CP violation in the standard model is insufficient to reproduce the observed value. Other models have been suggested, mostly based on an extension of the scalar structure. In that case, the effects are large enough, but the CP-violation parameters appearing in the evaluation of the baryon asymmetry happen to be completely independent of those responsible for CP violation at low energy in the K system. LR models, however, in particular in the case of spontaneous P and CP violation, offer the desired connection[13][14].

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