Can we tell 4 from 2 ?

(components)

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Outline

- Who thought neutrinos should be massless?
- Neutrino masses: Majorana, Dirac …
- Can we tell 4 from 2 components ?
  - Neutrino-Antineutrino oscillations ??? (no!)
  - Cosmology?
  - Magnetic moments ? - a new inequality

Additional material (ask me during breaks ..)

- Oscillations - the polarized light analogy (demonstration)
- free neutrinos vs neutrinos in matter
- Mass patterns … a challenging model
- R neutrinos put to use : leptogenesis – falsifiable by light W_R
- R neutrinos as Dark Matter and detection with light W_R
- For fun… neutrino lensing

Are neutrinos different ?

- Masses are very small (one could even vanish) ;
  we only know the differences of their squares.

- « Cabibbo » mixing is important, might even be more complicated
  (extra phases if Majorana, mixing with steriles)

- We don't even know the number of degrees of freedom (Majorana vs Dirac)

- They violate the ***separate*** conservation of electron, muon and tau numbers

Facts

Conjectures

- *They might violate the* ***global*** *lepton number (neutrinoless double beta)*

- *they could explain the Defeat of Antimatter (leptogenesis)*

- *They suggest (via See-Saw or other) the presence of new particles,*
  new scales, and could even accomodate extra dimensions

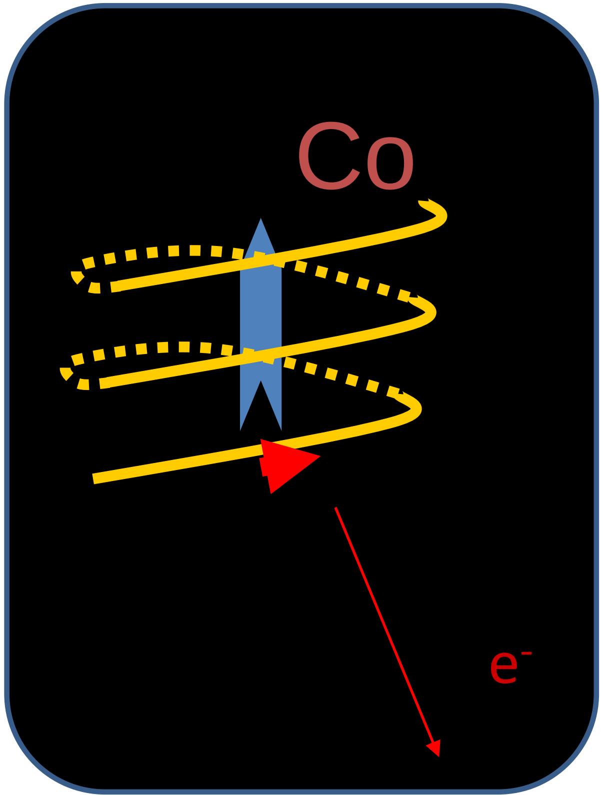
They pester us with re-learning about
Dirac, Majorana, degrees of freedom, oscillations, …

while the rest of the fermions seem so simple
by comparison!

Should neutrinos have been massless for the Standars Model?

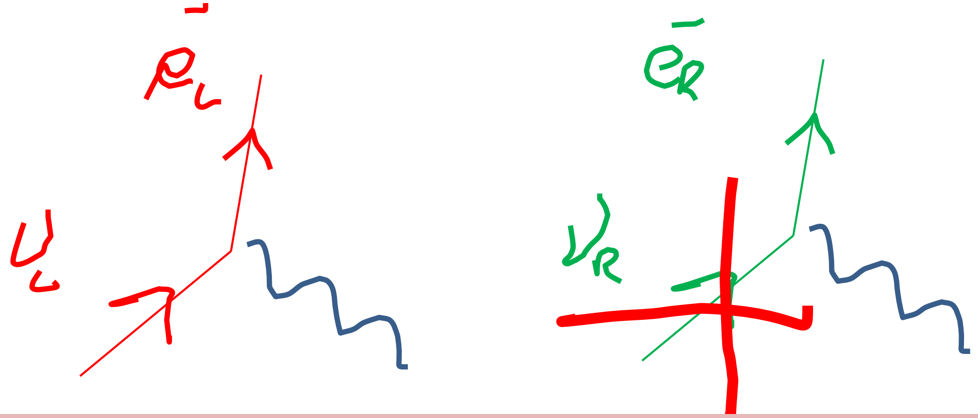
NO!

Once upon a time (has it completely ended?) people used to blame P violation on the absence of right-handed neutrinos …



P violation was clearly demonstrated in the Wu experiment ..

It is easy to explain if only left-handed electrons are produced in a charged vector current.



Killing the right-handed neutrino allows for parity violation in charged currents, even if the coupling is pure vector

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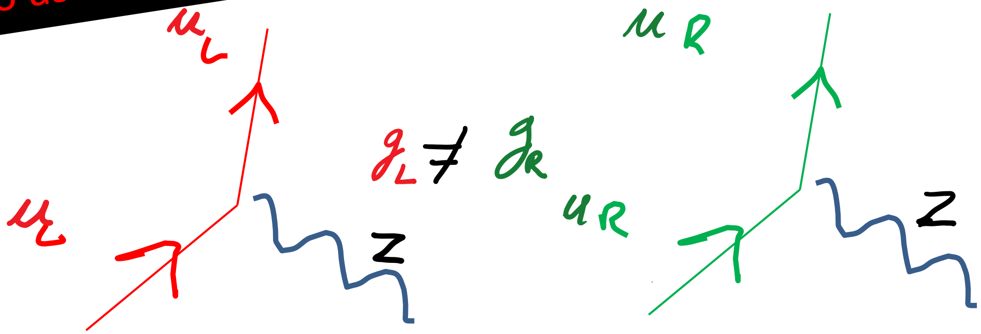
This was NEVER a solution …Assuming the whole world to be symmetrical under P, and taking the right-handed neutrino as the BAD GUY was NO SOLUTION.



- Not a solution today : we know the the Standard Model has neutral currents which violate P (parity violation in atoms, asymmetrical couplings of Z to quarks ..
- Even at the time of Wu's experiment, it was not a solution … this experiment was only a confirmation, a demonstration of P violation, known from the K→ 2 π and K→ 3 π (the Θ τ puzzle) where neutrinos don't play!

Still, in a way the doublet (ν_L e_L) was at the basis of the Standard Model, but the actual symmetry was experimentally found to be $SU(2)_L$, applied to all known fermions, including quarks

From the «absence of ν_R » to « massless neutrinos »

The «absence of ν_R » meant that « ordinary » (Dirac) masses were excluded …

This fitted well the fact that very small neutrino masses (at least for the electron neutrino) were requested from β decay kinematics.

…and this lead to the legend that neutrinos had to be massless in the Standard Model
In fact, masses were simply omitted in the first version
(which also lacked quarks, families, CP violation..)

But .. Evidence for neutrino masses!

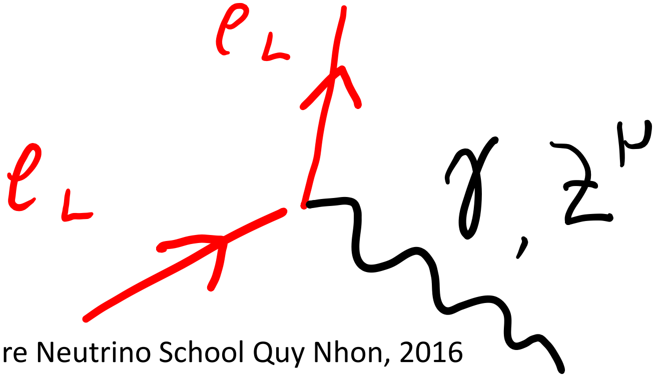
For questions of language, it is easier to speak of the electron + positron…

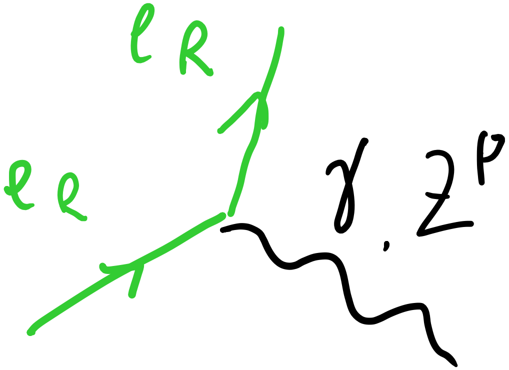
$$\begin{pmatrix} e_L \\ e_R \end{pmatrix} = \begin{pmatrix} e_{L1} \\ e_{L2} \\ e_{R1} \\ e_{R2} \end{pmatrix}$$

The Dirac spinor breaks down into 2 « Weyl » spinors,

$$\begin{pmatrix} \xi_L \\ \eta_R \end{pmatrix}$$

Gauge interactions talk separately to the L (left-handed) and R (right -handed)

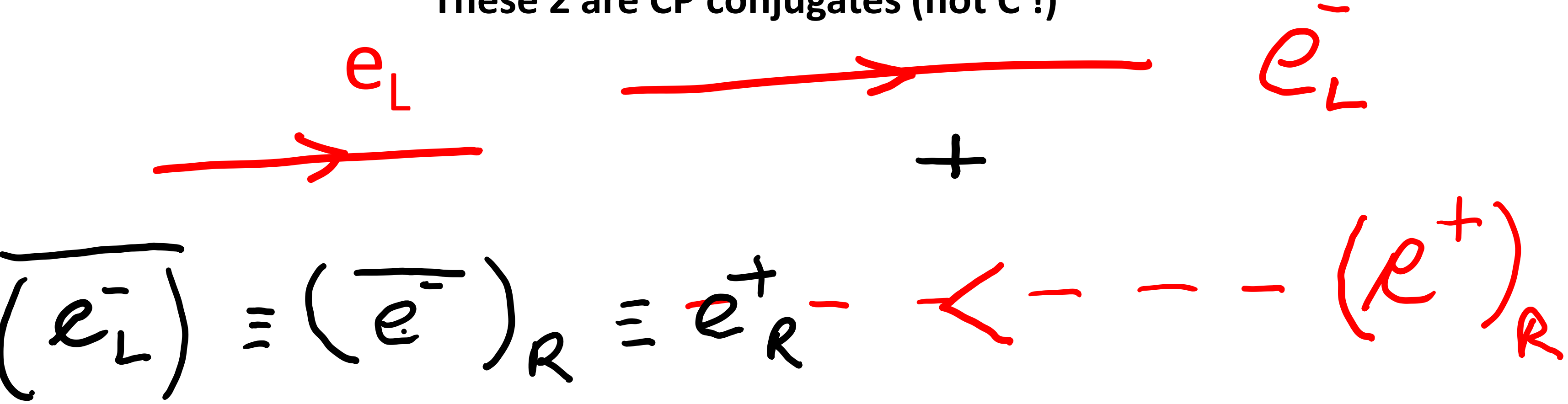




$$\begin{pmatrix} e_L \\ e_R \end{pmatrix}$$

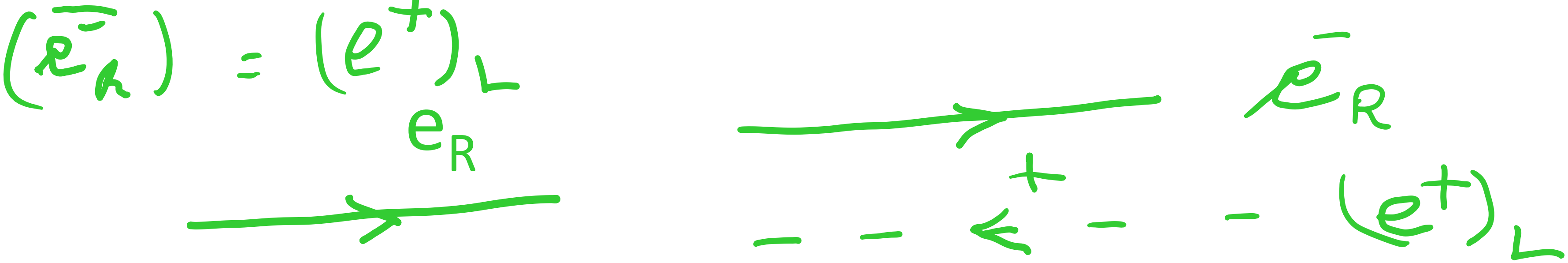
e_L Describes 2 things : the destruction of a L-handed electron and the creation of a R-handed positron

We can choose to use the electron or the positron for our description
These 2 are CP conjugates (not C !)

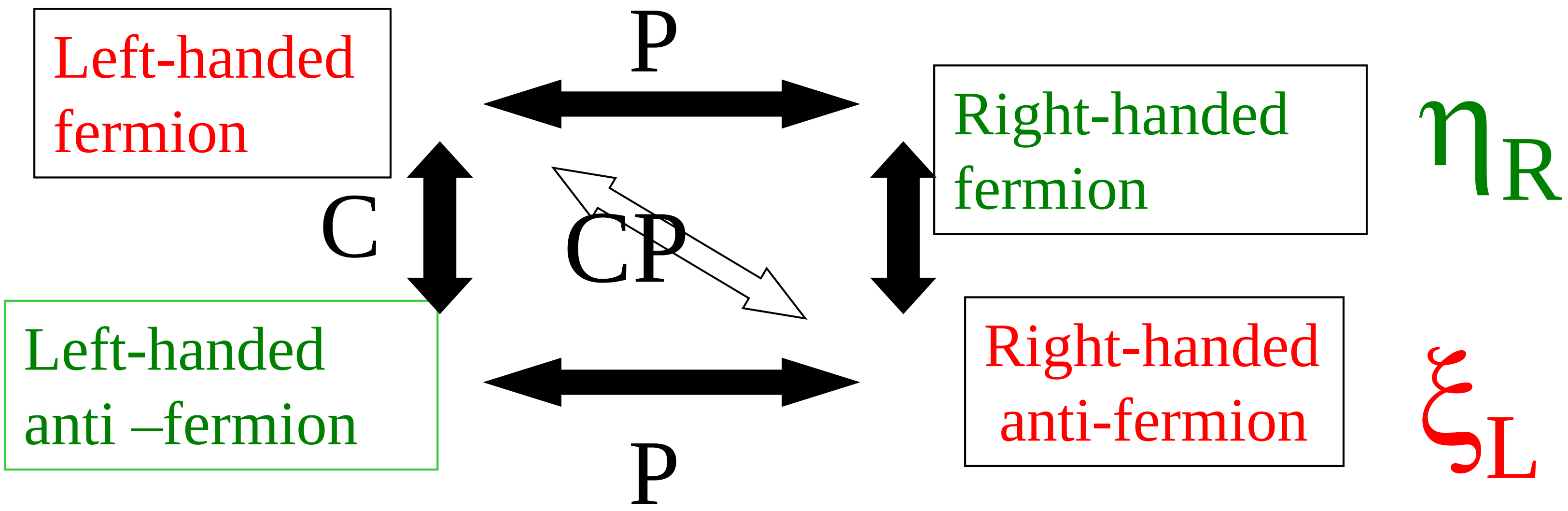


$$\overline{(e^-_L)} \equiv (\overline{e^-})_R \equiv e^+_R$$

But e_L does not describe the other 2 states ..



$$\overline{(e^-_R)} = (e^+)_L$$



Left-handed fermion

P

Right-handed fermion η_R

C

CP

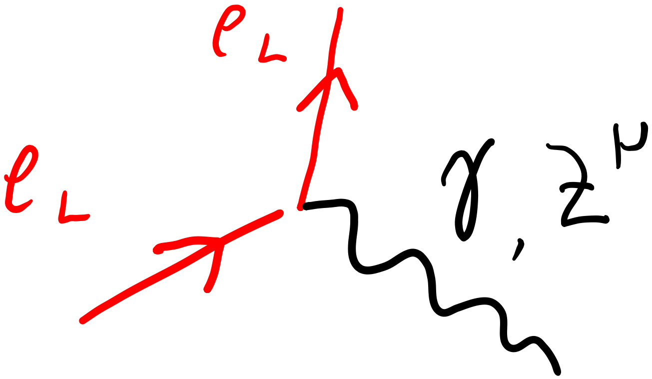
Left-handed anti –fermion

P

Right-handed anti-fermion ξ_L

The simplest coupling only introduces the left-handed Weyl spinor,
C and P are violated, but CP is conserved : this is THE symmetry of gauge interactions,

ξ_L



How can we write a mass term ?

A « mass » term must be invariant under proper Lorentz transformations (but we don’t impose P or C, which are broken in the SM.

Equations of motion must lead to

$$p^2 = |m|^2$$

W e introduce here 2 spinors,

We assume both to be L,

(if not, perform a CP transformation)

$$\begin{pmatrix}\psi_L\\ 0\end{pmatrix} = \begin{pmatrix}\psi_{L1}\\ \psi_{L2}\\ 0\end{pmatrix} \; ; \; \begin{pmatrix}\xi_L\\ 0\end{pmatrix}$$

… this is just the spin singlet !

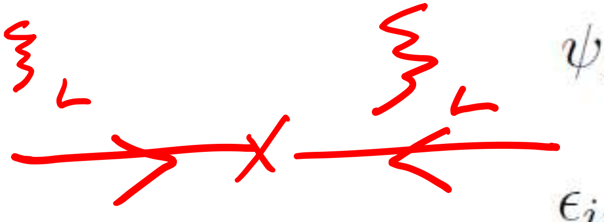
$$\uparrow\downarrow - \downarrow\uparrow$$

The Lorentz invariant then reads NEED to form a SCALAR term for the mass in the Lagrangian

$$\psi_{L1}\xi_{L2} - \psi_{L2}\xi_{L1} = \epsilon_{ij} \quad \psi_{Li}\xi_{Lj}$$

This expression covers ALL cases! (Majorana+Dirac)

$$\psi_{L1}\xi_{L2} - \psi_{L2}\xi_{L1} = \epsilon_{ij}\ \ \psi_{Li}\xi_{Lj}$$



$$\psi_L = \xi_L$$

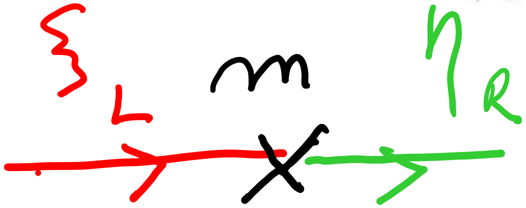
$$\epsilon_{ij}\xi_{Li}\xi_{Lj}$$

Creates (or destroys) 2 units
of fermionic number :
« Majorana mass term»

2 special cases :

$$\psi_{Li} \quad = \quad \epsilon_{ik}\overline{\eta_{Rk}}$$

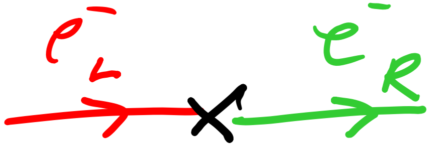
« Dirac mass term »



$$m\ \overline{\eta_{Rk}}\xi_{Lk}$$

If we can assign **the same fermionic number to η and ξ,**
Fermion number is now conserved

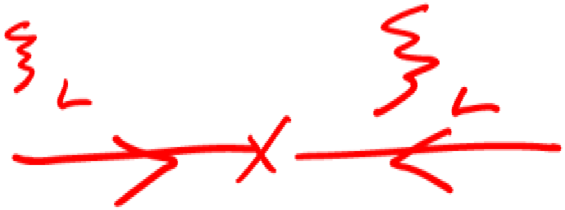
For the electron, only the « Dirac » mass term is allowed – the « Majorana » one does not even conserve electric charge!

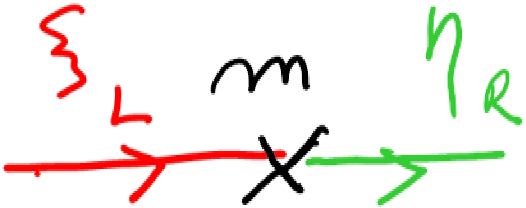


On the other hand, for the neutrino, charge is not a problem, and we can use the « Majorana » mass. It violates leptonic number, but if the mass is small enough, this escapes detection.

It is thus possible to have Neutrino masses without introducing the right-handed neutrino

The sign (or phase) of the mass.





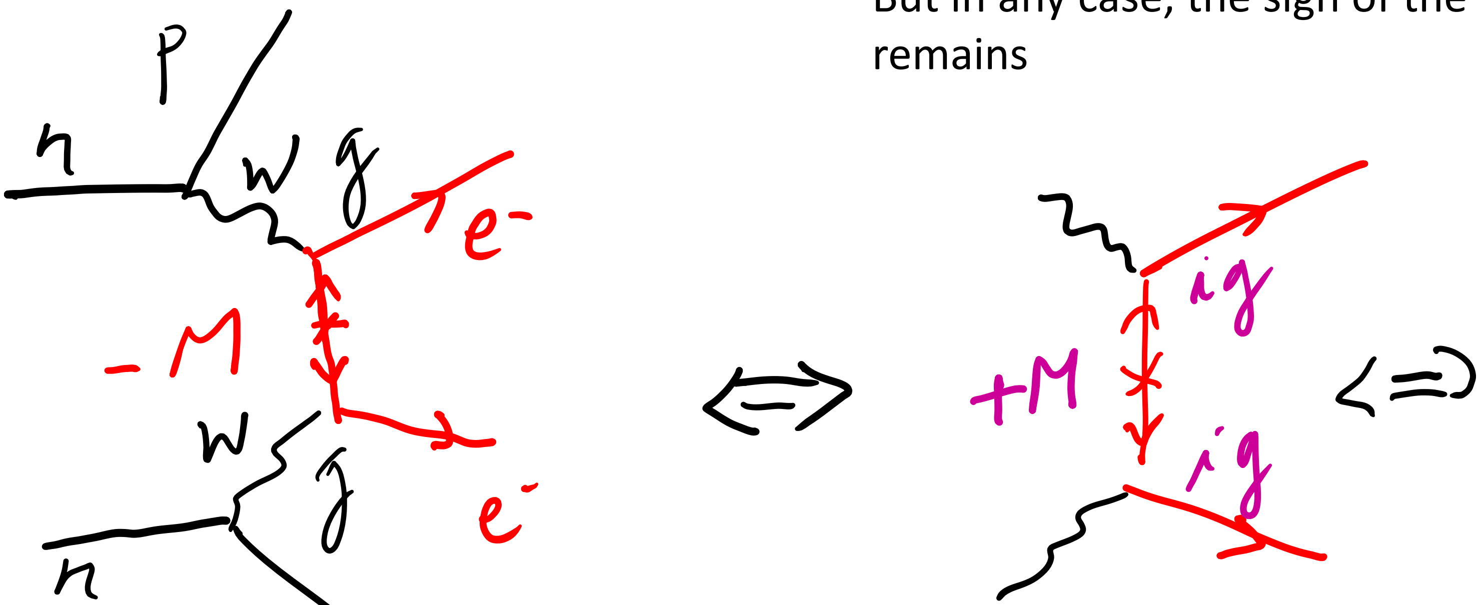
The parameter m in the Lagrangian
is in general a complex number.
In the case of one family, in the Dirac case, we can always re-define m to be real, just by changing the sign of η_R , which does not couple to anyone.

The sign of the fermion mass – Majorana case

$$-M \quad \epsilon_{ij}\xi_{Li}\xi_{Lj}$$

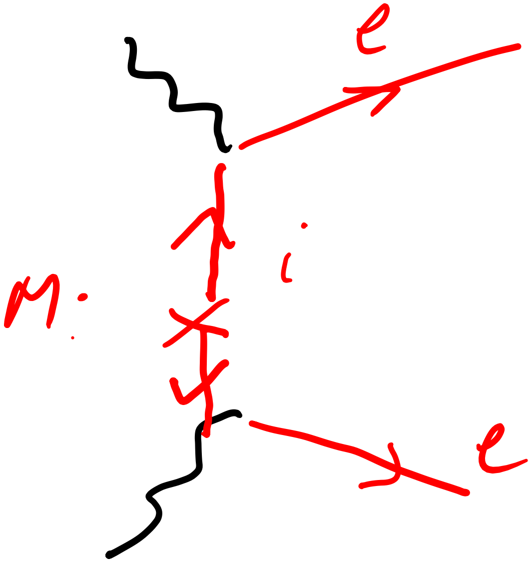
Here, we cannot re-define the sign of the mass without affecting the interactions … we can bring m to be real by re-defining $\xi \to i\,\xi$

But in any case, the sign of the amplitude remains



$$\frac{-Mg^2}{q^2-M^2}$$

Neutrinoless Double Beta decay is sensitive to the weighted sum of masses, including Majorana phases



$$\sum \left(\frac{M_i}{q^2 - M_i^2} \cdot g^2 V_{ei}^2 \right)$$

$$\sum M_i = 0$$

Special case : for one flavor,
Dirac can be seen as 2 semi-spinors with
equal but opposite masses and equal couplings

For later use : the cancellation occurs not in one family, but across families
« Pseudo-Dirac »

This far we spoke of Weyl neutrino, Majorana mass terms, but not of Majorana spinors… In fact, they are not needed in 3+1 dim … just another (confusing) notation

$$\psi = \begin{pmatrix} \lambda \\ \rho \end{pmatrix}$$

$$\psi^c = \psi$$

$$\Big\Downarrow$$

$$\lambda_i = \epsilon_{ij}\ \rho^{+t}$$

Majorana or Weyl spinors ?

In 4-D : equivalent, Majorana is just a REDUNDANT way to write WEYL spinors

$$\begin{pmatrix} \psi_L \\ \\ (\psi_L)^c_R \end{pmatrix}$$

This is not true in more dimensions!

Exercise: A Dirac spinor can indeed be seen as the sum of 2 Majorana spinors of equal and opposite masses ..

$$m \ \bar{\Psi} \ \Psi$$

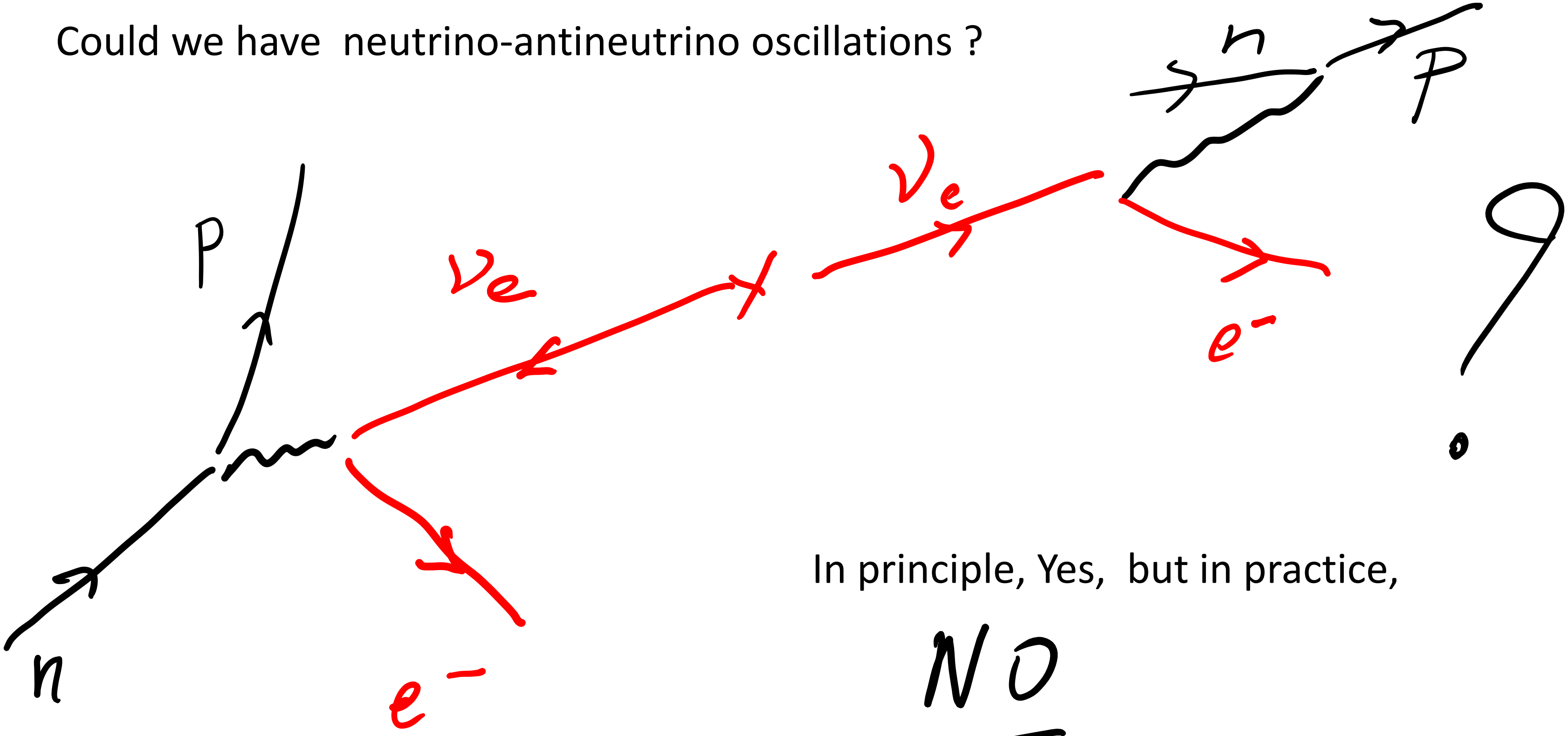
$$\chi = \frac{1}{\sqrt{2}}(\Psi + \Psi^c)$$

$$\lambda = \frac{1}{\sqrt{2}}(\Psi - \Psi^c)$$

$$\frac{m}{2}\bar{\chi}^c\chi - \frac{m}{2}\bar{\lambda}^c\lambda$$

Beyond the Neutrinoless Double beta decay,
Can we probe the Majorana nature of neutrino masses?

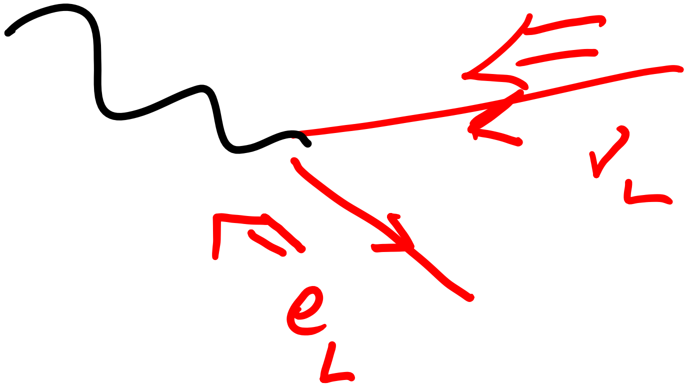
Could we have neutrino-antineutrino oscillations ?



In principle, Yes, but in practice,

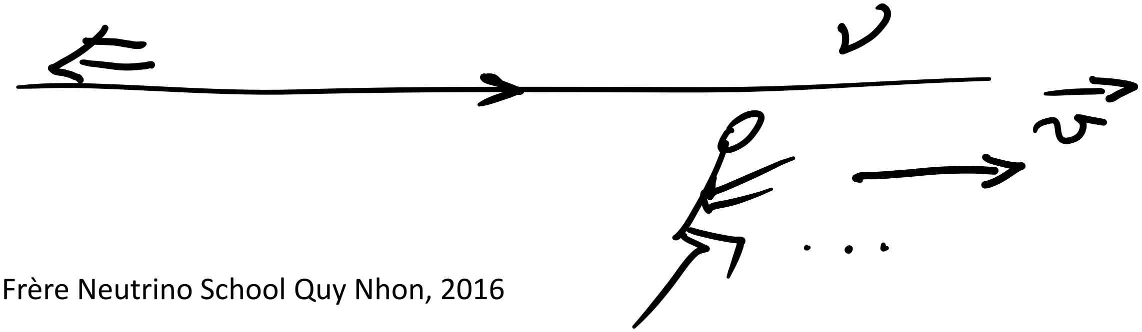
NO

Even though the lepton number is not conserved, angular momentum suppresses this reaction

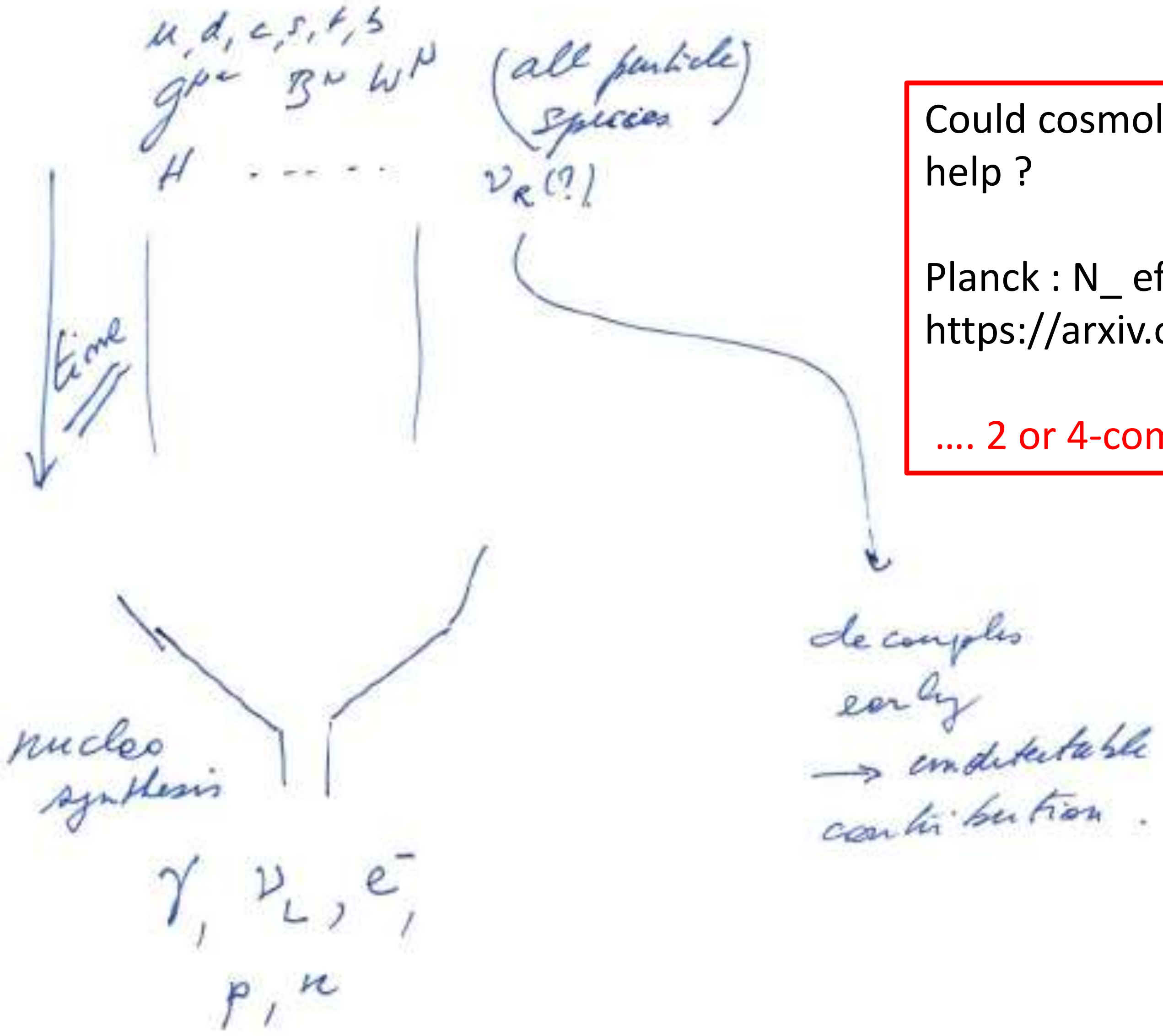


The ν_L stays linked to e^-_L, and not to e^+_R by the W's in the SM

As long as the detector and emitter don't have large relative speeds (in comparison to the neutrino), helicity is conserved up to factor of m/E in amplitude Even for 1MeV neutrinos, this gives a suppression of 10^{-12} in probability



Could the cosmological counting of neutrinos help us ?



Could cosmological neutrino counting help ?

Planck : N_ eff = 3.15 +/- 0.23
https://arxiv.org/abs/1502.01589

…. 2 or 4-components ? … not sensitive!

Magnetic moments?

For ONE Weyl neutrino, a magnetic moment is forbidden by Fermi statistics ..

Is it a way to exclude Majorana masses? ↑↑

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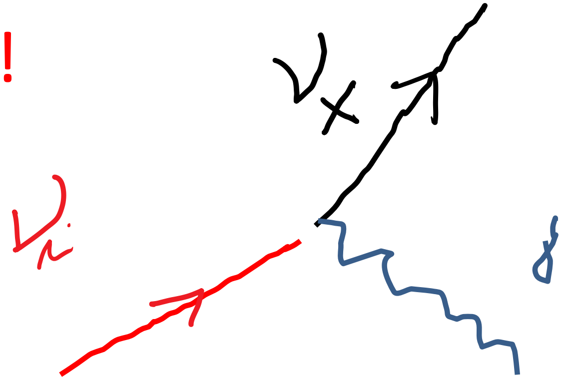
For ONE Weyl neutrino, a magnetic moment is forbidden by Fermi statistics ..

Is it a way to exclude Majorana masses? ↑↑

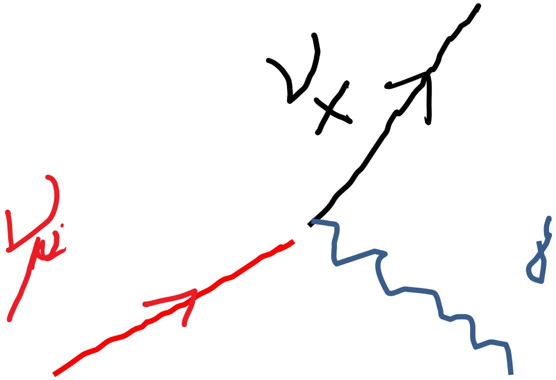
NO, TRANSITION magnetic moments are still allowed …

and undistinguishable!



$$H_{\rm eff} = \frac{\mu_{IJ}}{2}\overline{\nu_I^c}\sigma_{\alpha\beta}P_L\nu_J F^{\alpha\beta} + \text{h.c.},$$

In Weyl basis



effective moment for ν_X

$$\sqrt{|\mu_{e\mu}|^2 + |\mu_{\tau\mu}|^2}\left(\overline{\nu_X^c}\sigma_{\alpha\beta}\nu_\mu F^{\alpha\beta}\right),$$

$$\overline{\nu_X^c} \equiv \frac{\left(\mu_{e\mu}\overline{\nu_e^c} + \mu_{\tau\mu}\overline{\nu_\tau^c}\right)}{\sqrt{|\mu_{e\mu}|^2 + |\mu_{\tau\mu}|^2}}.$$

Effective electromagnetic moment for the muon neutrino
In WEYL (Majorana) case :

$$|\mu_{\nu_\mu}| \equiv \sqrt{|\mu_{e\mu}|^2 + |\mu_{\tau\mu}|^2}$$

Effective electromagnetic moment for the muon neutrino :

$$|\mu_{\nu_\mu}| \equiv \sqrt{|\mu_{e\mu}|^2 + |\mu_{\tau\mu}|^2}$$

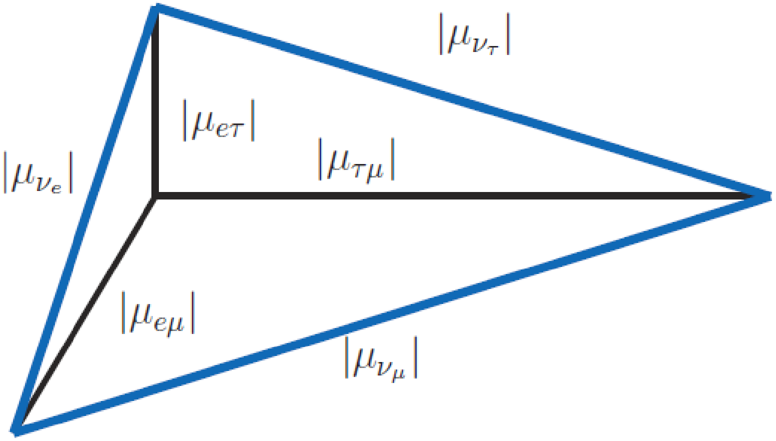


Figure 1: $|\mu_{\nu_J}|$ forms a right triangle with $|\mu_{IJ}|$ and $|\mu_{KJ}|$ (for $I \neq J \neq K$). $|\mu_{\nu_{I,J,K}}|$ thus also form a triangle (shown in thick blue), in general not with right angles.

JMF, J Heeck, S Mollet arXiv:1506.02964 to appear in PRD

It is then easy to work out the inequalities ..

$$|\mu_{\nu_\tau}|^2 \leq |\mu_{\nu_e}|^2 + |\mu_{\nu_\mu}|^2 \,,$$
$$|\mu_{\nu_\mu}|^2 \leq |\mu_{\nu_\tau}|^2 + |\mu_{\nu_e}|^2 \,,$$
$$|\mu_{\nu_e}|^2 \leq |\mu_{\nu_\mu}|^2 + |\mu_{\nu_\tau}|^2 \,,$$

These are stronger than the more obvious « triangle inequalities »:
(none of the angles can be > 90°) $||\mu_{\nu_J}| - |\mu_{\nu_K}|| \leq |\mu_{\nu_I}| \leq |\mu_{\nu_J}| + |\mu_{\nu_K}|$

Current limits (terrestrial)

$$|\mu_{\nu_e}| < 2.9 \times 10^{-11} \mu_B \,, \qquad |\mu_{\nu_\mu}| < 6.8 \times 10^{-10} \mu_B \,, \qquad |\mu_{\nu_\tau}| < 3.9 \times 10^{-7} \mu_B \,.$$

Perspectives : SHiP (CERN SPS) could improve considerably the τ neutrino limit …

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Current limits (astrophysics – in fact sum over all neutrinos)

$$4.5 \times 10^{-12} \mu_B$$

Hopeless for terrestrial mesurements?
NO …

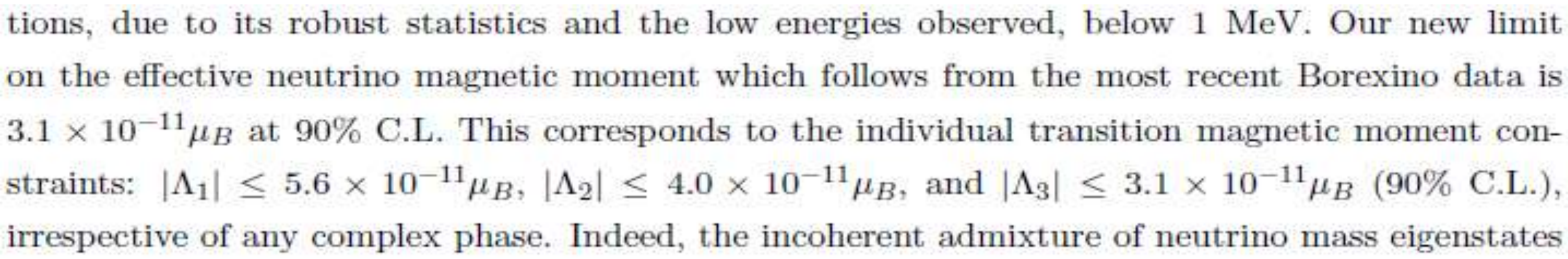
if there is a 4th light (sterile) neutrino, with mass > keV,
astro limits don’t apply
and a large electromagnetic moment could be observed … SHiP is in business !

(by the way, light extra neutrinos are considered as components of Dark Matter…)

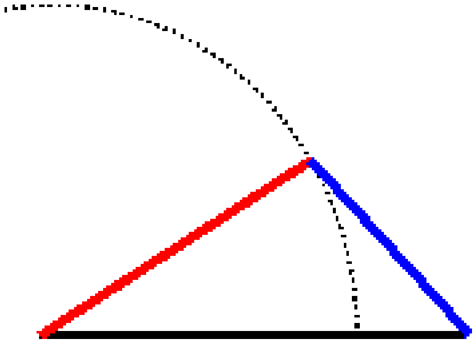
Updating neutrino magnetic moment constraints B.C. Canas, O.G. Miranda, A. Parada ,M. Tortola, Jose W.F. Valle Phys.Lett. B753 (2016) 191-198, Addendum: Phys.Lett. B757 (2016) 568-568 arXiv:1510.01684 (**an update of** : Constraining Majorana neutrino electromagnetic properties from the LMA-MSW solution of the solar neutrino problem W. Grimus, M. Maltoni, T. Schwetz, M.A. Tortola, J.W.F. Valle Nucl.Phys. B648 (2003) 376-396 hep-ph/0208132)

Update ….

Borexino brings interesting new bounds (from « oscillated » Solar neutrinos)



tions, due to its robust statistics and the low energies observed, below 1 MeV. Our new limit on the effective neutrino magnetic moment which follows from the most recent Borexino data is $3.1 \times 10^{-11}\mu_B$ at 90% C.L. This corresponds to the individual transition magnetic moment constraints: $|\Lambda_1| \leq 5.6 \times 10^{-11}\mu_B$, $|\Lambda_2| \leq 4.0 \times 10^{-11}\mu_B$, and $|\Lambda_3| \leq 3.1 \times 10^{-11}\mu_B$ (90% C.L.), irrespective of any complex phase. Indeed, the incoherent admixture of neutrino mass eigenstates



Using these numbers, we have (if we saturate the bounds)
31.36 > 16 + 9.61 …. is there hope to improve and
get an actual check at the 10^{-11} level?

Still --- complicated models needed for large magnetic moments !!!
For instance ..

« Double see-saw »

2 singlets

$$M_\nu = \begin{array}{c} \begin{array}{ccc} \nu_L & \nu_R & \nu_S \end{array} \\ \begin{pmatrix} 0 & m & 0 \\ m^{T} & 0 & M^{T} \\ 0 & M & m_\sigma \end{pmatrix} \end{array}$$

$m = \lambda v$

λ can then be large, and lead to observable effets, since the light neutrino mass is proportional to m_σ

$$m_{\nu_1} \approx (m/M)^2 m_\sigma , \qquad m_{\nu_{2,3}} \approx M \pm m_\sigma/2 ,$$

(remark : this is an example of « pseudo-Dirac »,

since $\nu_R + \nu_S$ act as a Dirac pair, whose contributions to the light neutrino compensate.

(an old idea, .. Langacker, Mohapatra, Antoniadis, 1986-88, jmf+Liu, recently revived…)

Additional slides FOR FUN

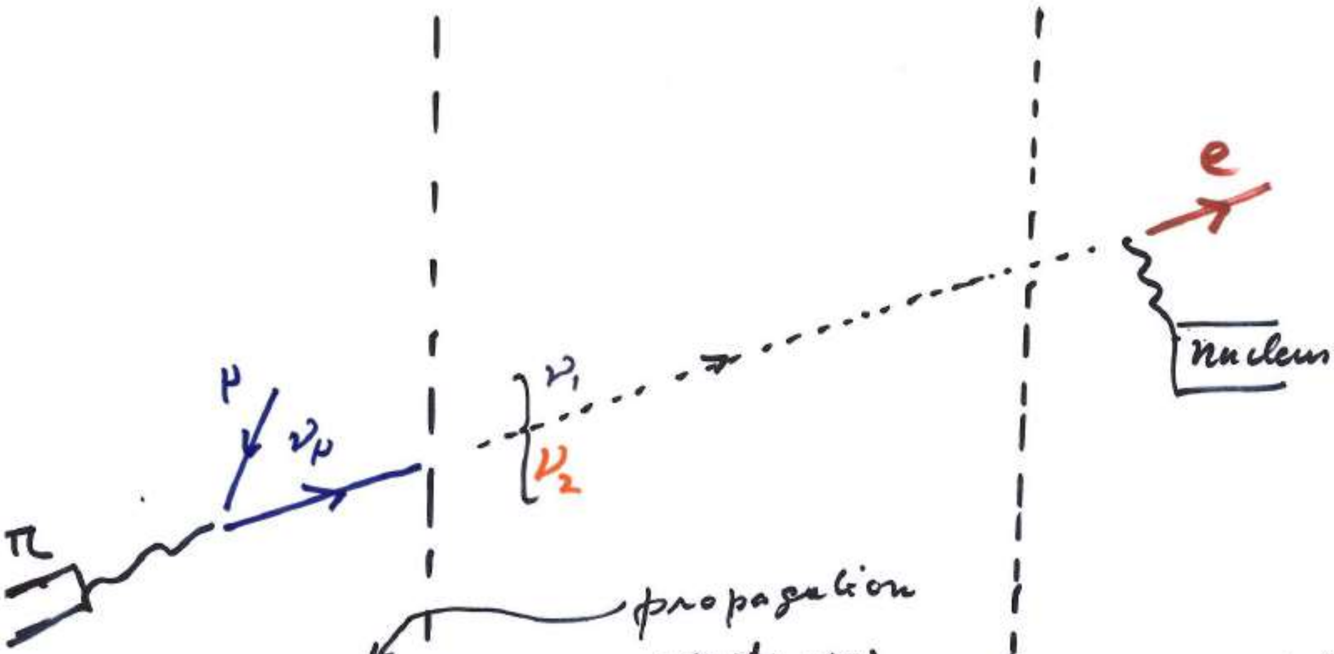
Neutrino Oscillations – polarized light analogy --
a practical exercise

Some funny possibilities with neutrinos
(neutrino lensing, neutrinos as dark matter

Finding a W_R at a collider near you would invalidate leptogenesis..

A model based on 6 dimensions, which predicted θ_{13}

But .. Evidence for neutrino masses! (?)
Neutrino oscillations prove that the « propagation states » are different from the « creation » and « detection » states.



$$|\nu_\mu\rangle = \cos\theta\,|\nu_1\rangle\, e^{iE(t - v_1 x)} + \sin\theta\,|\nu_2\rangle\, e^{iE(t - v_2 x)}$$

if $v_2 \neq v_1$ emerging state is no longer ν_μ

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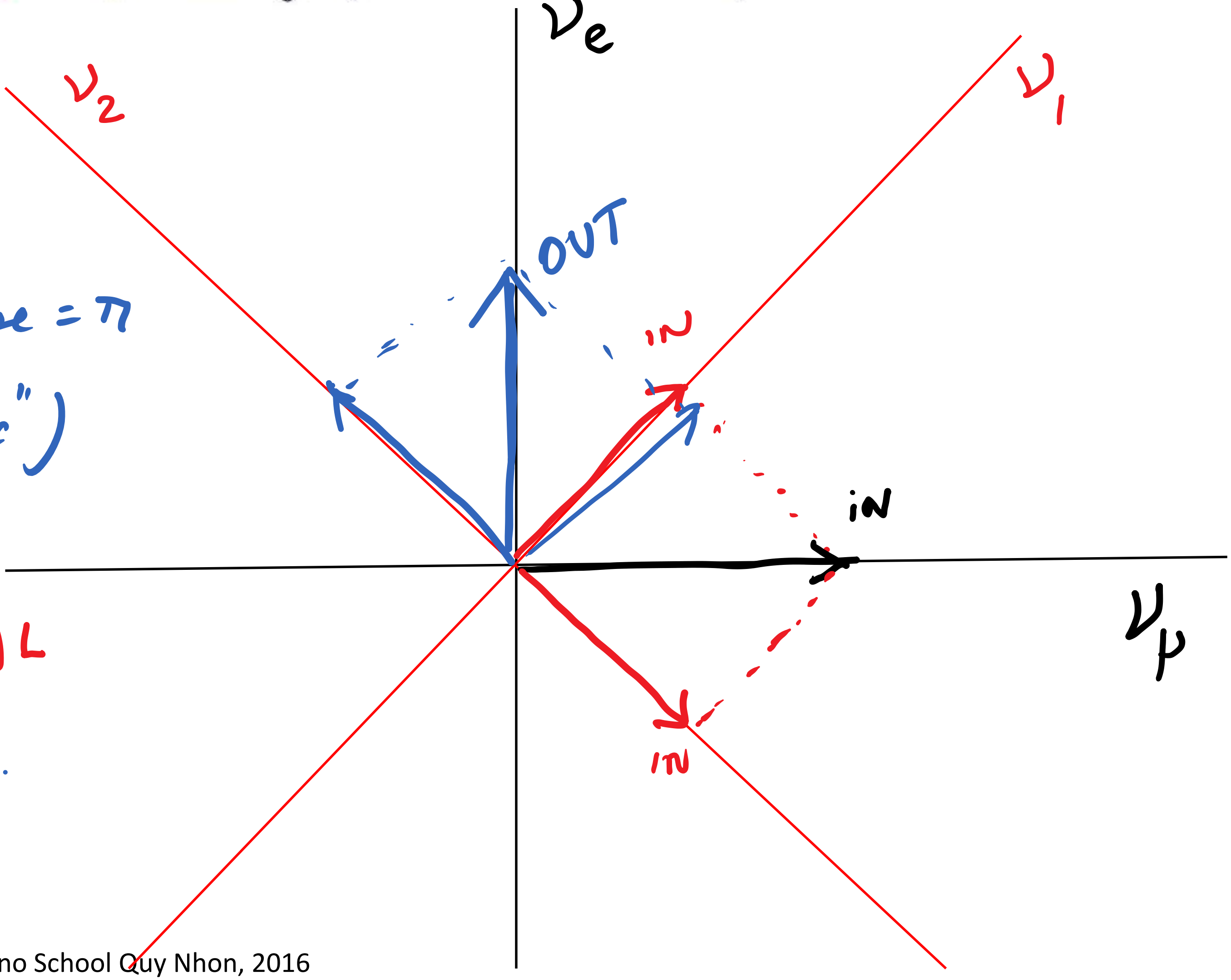
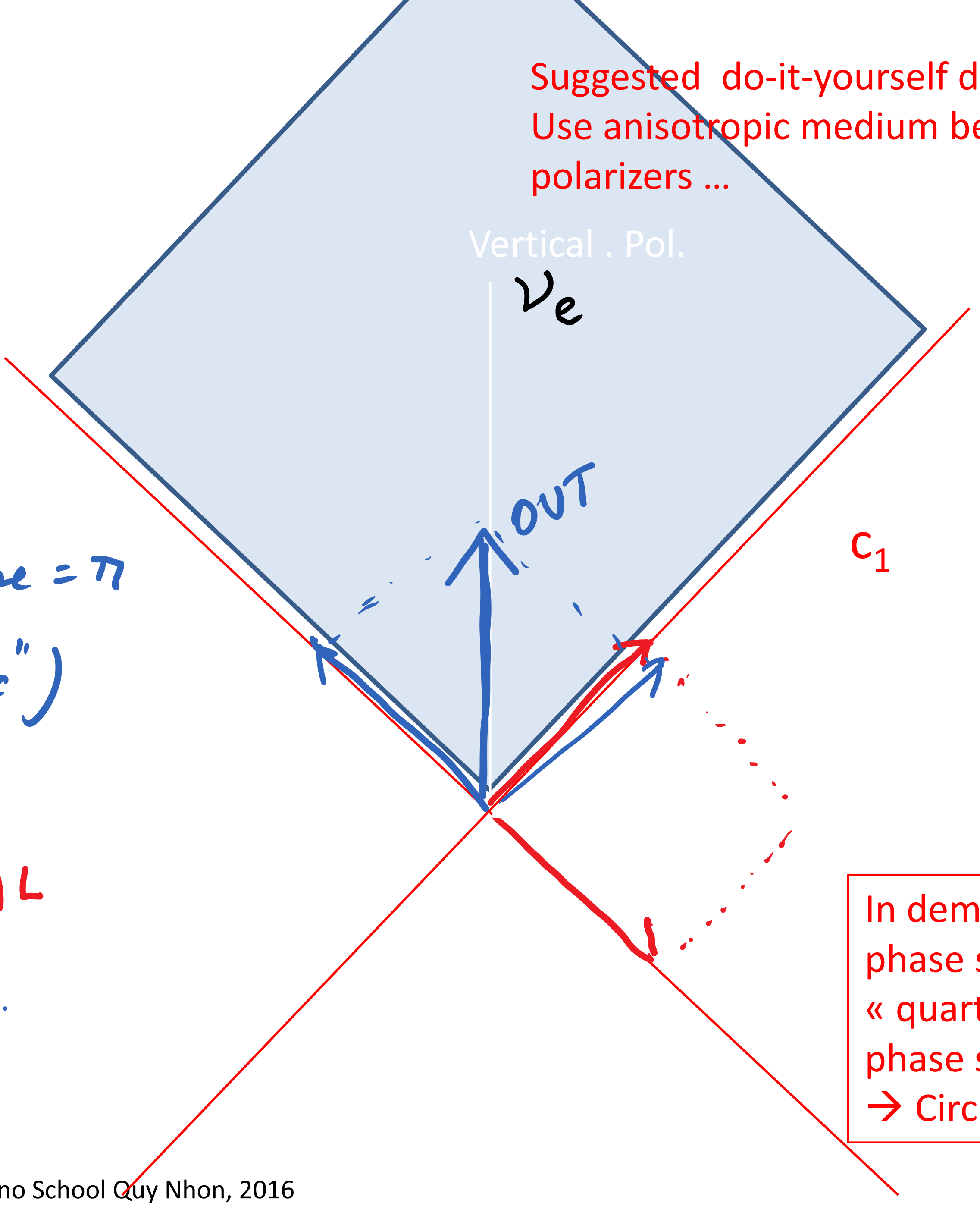


(here, Δ phase $= \pi$
"half wave")

$e^{iE(v_1-v_2)L}$

Suggested do-it-yourself demo:
Use anisotropic medium between crossed polarizers …



(here, Δphase $= \pi$
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$e^{iE(\nu_1-\nu_2)L}$

In demo, use half wave plate,
phase shift = π (here) or
« quarter wave plate »,
phase shift = $\pi/2$
→ Circular polarization,

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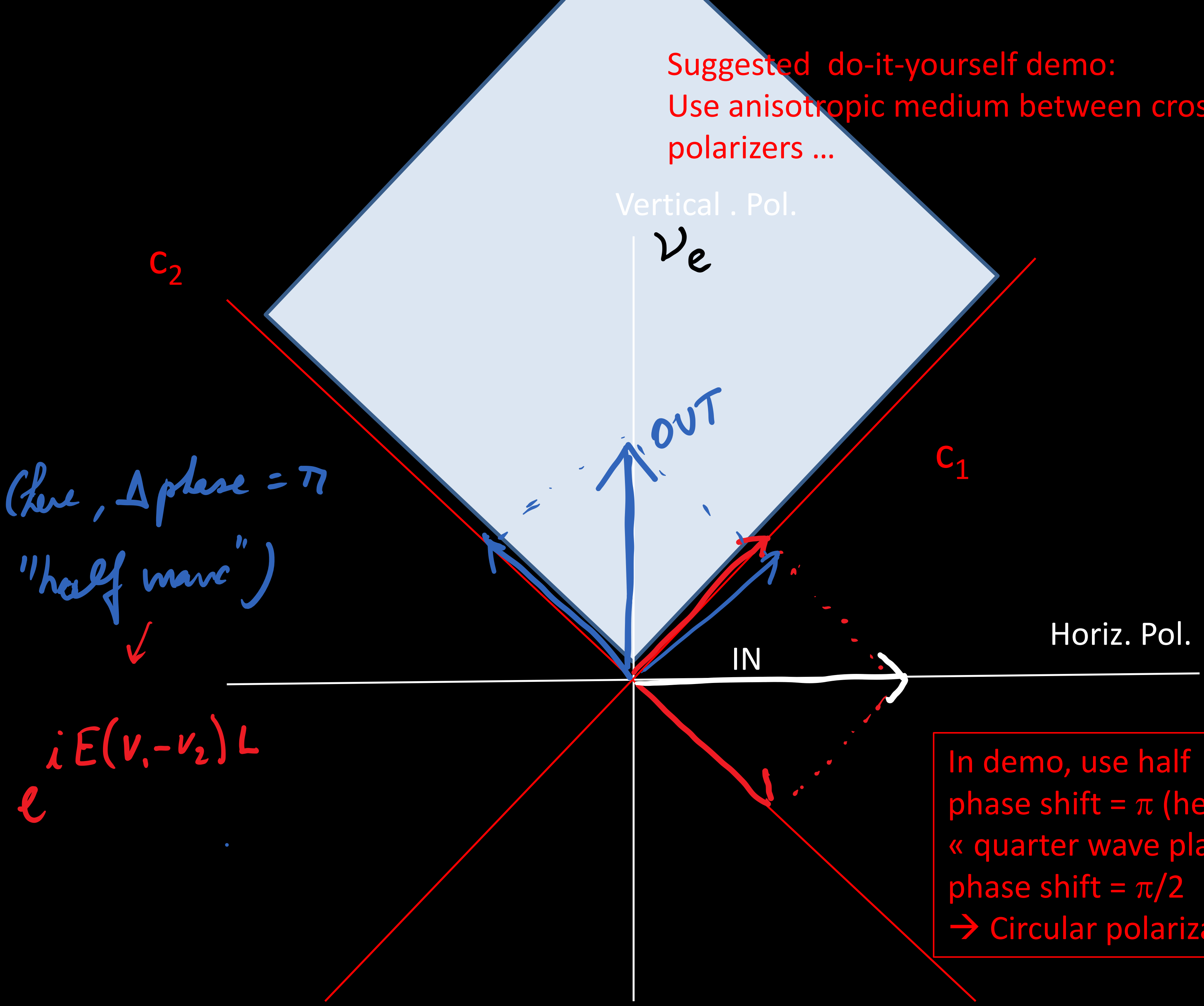
Use anisotropic medium between crossed polarizers …



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$$e^{iE(v_1-v_2)L}$$

In demo, use half wave plate,
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phase shift = π/2
→ Circular polarization,

Why would be the propagation speed of neutrinos 1 and 2 differ?

It could be MASS,

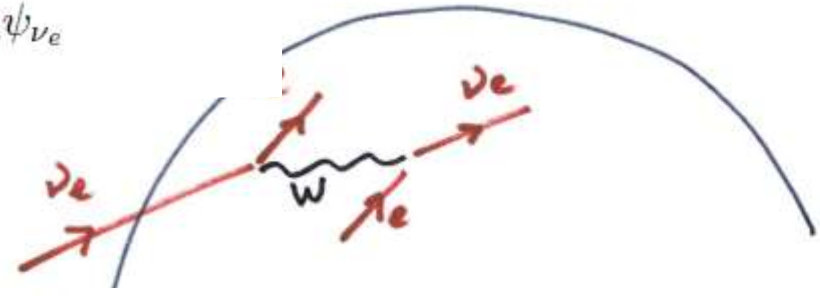
$$\begin{aligned} E^2 &= \vec{p}^{\,2} + m^2 \\ v &= |\vec{p}|/E \\ v &= \sqrt{1-(m/E)^2} \\ (v_1 - v_2)\,L &= \frac{(m_2^2 - m_1^2)\,L}{2E} \end{aligned}$$

The effect is the same for neutrinos and antineutrinos, does not depend on the type of mass (Majorana or Dirac)

But also any kind of interaction affecting differently 1 and 2
Well-known example : MSW effect

But also any kind of interaction affecting differently 1 and 2
Well-known example : MSW effect

$$\begin{aligned} Lagrangian &\supset \overline{\psi_{\nu_e}}(p^0\gamma^0 - \vec{p}\,\vec{\gamma})\psi_{\nu_e} - V \\ V &\supset \kappa G_F \overline{\psi_{\nu_e}}\gamma^\mu\psi_e \; \psi_e\gamma_\mu\psi_{\nu_e} \end{aligned}$$



After Fierzing,

$$\begin{aligned} \kappa G_F \overline{\psi_{\nu_e}}\gamma^\mu\psi_e \; \psi_e\gamma_\mu\psi_{\nu_e} &= \kappa' G_F \overline{\psi_{\nu_e}}\gamma^\mu L\psi_{\nu_e}\psi_e \; \psi_e\gamma_\mu L\psi_e \\ &= \kappa'' \overline{\psi_{\nu_e}}\gamma^0 L\psi_{\nu_e} G_F \rho_e \end{aligned}$$

ν_e, ν_μ
$\nu_{sterile}$
have ≠ interactions
in matter.

This means that we simply replace $(p^0)^2 - \vec{p}^{\,2} = m^2$ by

$$p^0 \to p^0 - \kappa'' G_F \rho_e$$

$$E^2 - 2p^0\kappa'' G_F - \vec{p}^{\,2} = m^2$$

And get an effective mass ..
which differs for neutrino
and antineutrino (CPT violation …
we interact with MATTER

$$m^2 \to m^2 + 2p^0\kappa'' G_F \rho_e$$

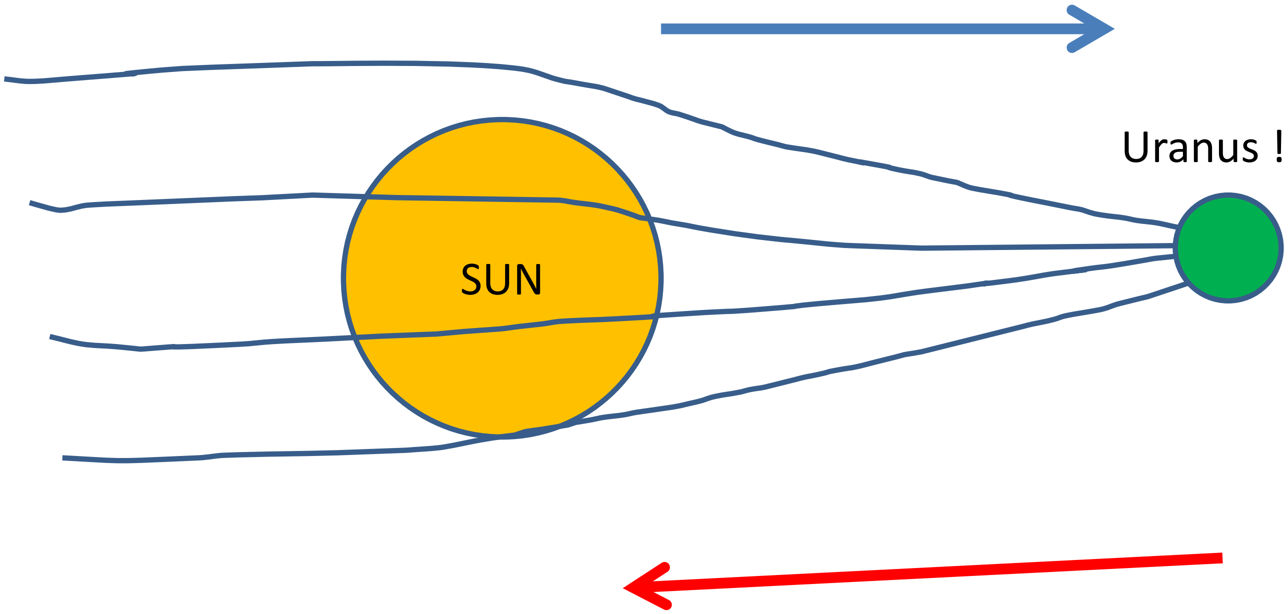
ν $p^0 > 0$

$\bar{\nu}$ $p^0 < 0$

Insights on neutrino lensing
R. Escribano jmf, D. Monderen, V. Van Elewyck (Brussels U.). **Phys.Lett. B512 (2001) 8-17**

Just for the fun .. Neutrino lensing…

Stars are Gravitational lenses but bad lenses for light,
But can be good lenses for neutrinos !



Exotica2

R. Escribano, J-M. F, D. Monderen, V. Van Elewyck
Phys.Lett. B512 (2001) 8-17

Also binary star as « neutrino light house »

Massive Neutrinos as dark matter:

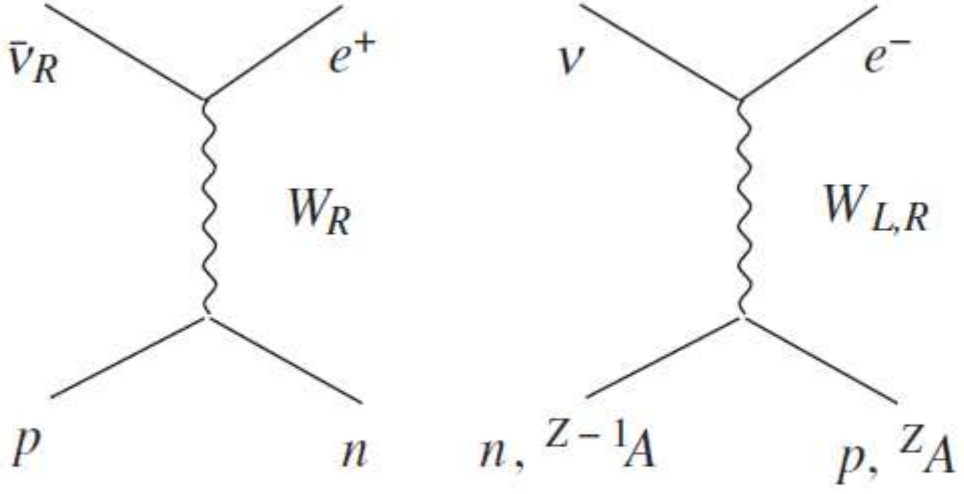
Could be constrained by Solar neutrino experiments …
DM has little momentum, but the mass of the heavy neutrino triggers the reaction.

If light W_R present and MeV « heavy neutrino »
Is a dark matter candidate,

we get limits from large underground detectors ..
Catalyse beta and betat+ decay

→ Limits .. For $m_R = 1$ MeV we obtain $MR/ML > 10–20$
JMF, L Lopez-Honorez, E Nezri, S Swillens, G Vertongen, **Phys.Rev. D75 (2007) 085017** hep-ph/0610240

Exotica 1



What are Right-handed neutrinos good for?

Heavy ν_R (= N) are found in grand unified theories like SO(10) and above,
But are specially usefull for inducing the DEFEAT OF ANTIMATTER

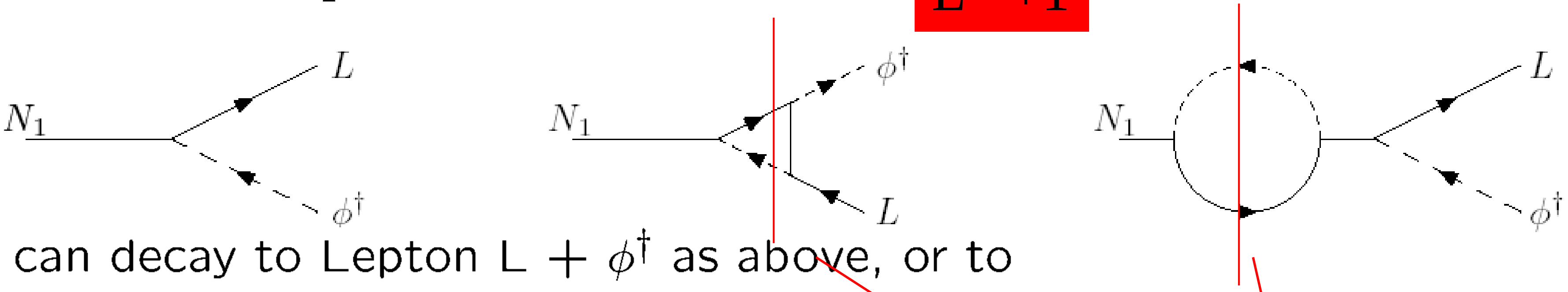
CP violating decay creates $L<0$, converted into $B>0$ by an anomaly-related mechanism (instantons)

How leptogenesis works….

Assume that we have some population of heavy N particles…
(either initial thermal population, or re-created after inflation) ; due to their heavy mass and relatively small coupling, N become easily relic particles.

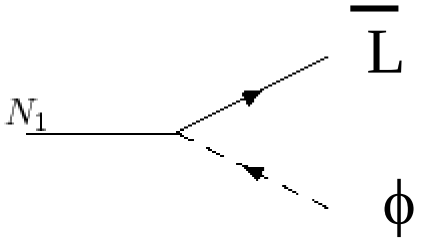
Generation of lepton number

L =+1



N can decay to Lepton L $+ \phi^\dagger$ as above, or to the opposite channel $\bar{L}\phi$

CP violation +
Interference term leads
to excess of L or anti-L



L =-1

Possible unitarity cuts

Constraints:

Heavy neutrinos must decay out of equilibrium

$$\tau(X) >> H^{-1}$$

$H = \dot{a}/a$ is the Hubble constant,

$$\tau^{-1} = \Gamma \cong g^2 M$$

$$H = \sqrt{g^*}\frac{T^2}{10^{19} GeV}$$

g^* is the number of degrees of freedom at the time

at decay : $T \approx M$,

Need enough CP violation;

for large splitting between neutrino masses, get

$$\varepsilon_i^{\phi} = -\frac{3}{16\pi}\frac{1}{\left[\lambda_\nu \lambda_\nu^\dagger\right]_{ii}}\sum_{j\neq i} \mathrm{Im}\left(\left[\lambda_\nu \lambda_\nu^\dagger\right]_{ij}^2\right)\frac{M_i}{M_j}.$$

Some rough estimations…

…What are the suitable values of λ and M?

Assume there is only one generic value of λ (in reality, a matrix)

$$\epsilon < \lambda^4/\lambda^2 \approx \lambda^2 > 10^{-8}$$

$$m_\nu = m^2/M \approx \lambda^2/M \approx .01eV$$

rough estimate of M scale (in GeV) needed…

similar to τ lepton →

λ	light neutrino .01 eV M ~	decay out of equil. M>	enough CP viol
.00001	10^7	10^8	need tuning
.0001	10^9	10^10	🟧
.001	10^11	10^12	🟩
.01	10^13	10^14	🟩
.1	10^15	10^16	🟩
1	10^17	10^18	large

At the difference of baryogenesis, the Yukawa matrix λ leaves a lot of freedom

Can leptogenesis be falsified ?

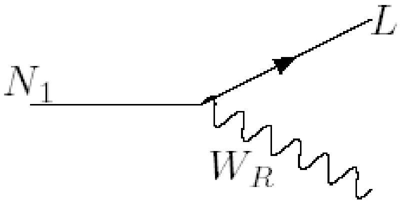
In general, no, since most mass ranges are unaccessible.
But .. Presence of ν_R suggest a larger symmetry, like SO(10) or $SU(2)_L \times SU(2)_R$

with the gauge inclusion

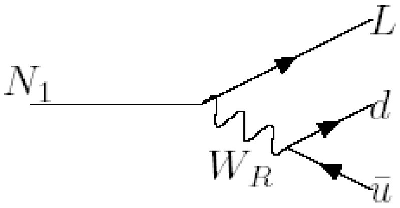
$$\epsilon_1 = \frac{\epsilon_1^0}{1+X}$$

diluted CP asymmetry

$$\underline{M_{W_R} < M_{N_1}}$$



$$\underline{M_{W_R} > M_{N_1}}$$

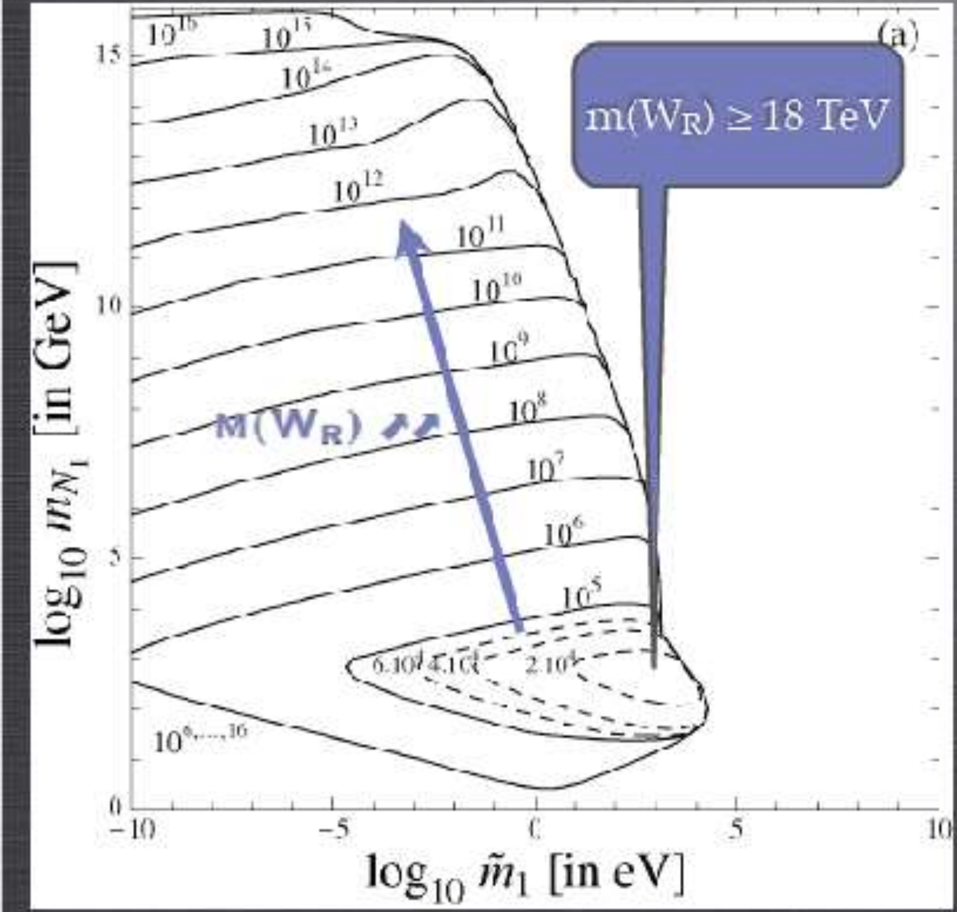


(competing effect : the presence of W_R allows a faster build-up of the N population after inflation)

S Carlier, JMF, FS Ling **Phys.Rev. D60 (1999) 096003**
JMF, T Hambye, G Vertongen **JHEP 0901 (2009) 051**

BOUNDS ON M(W_R) & M(N_R)

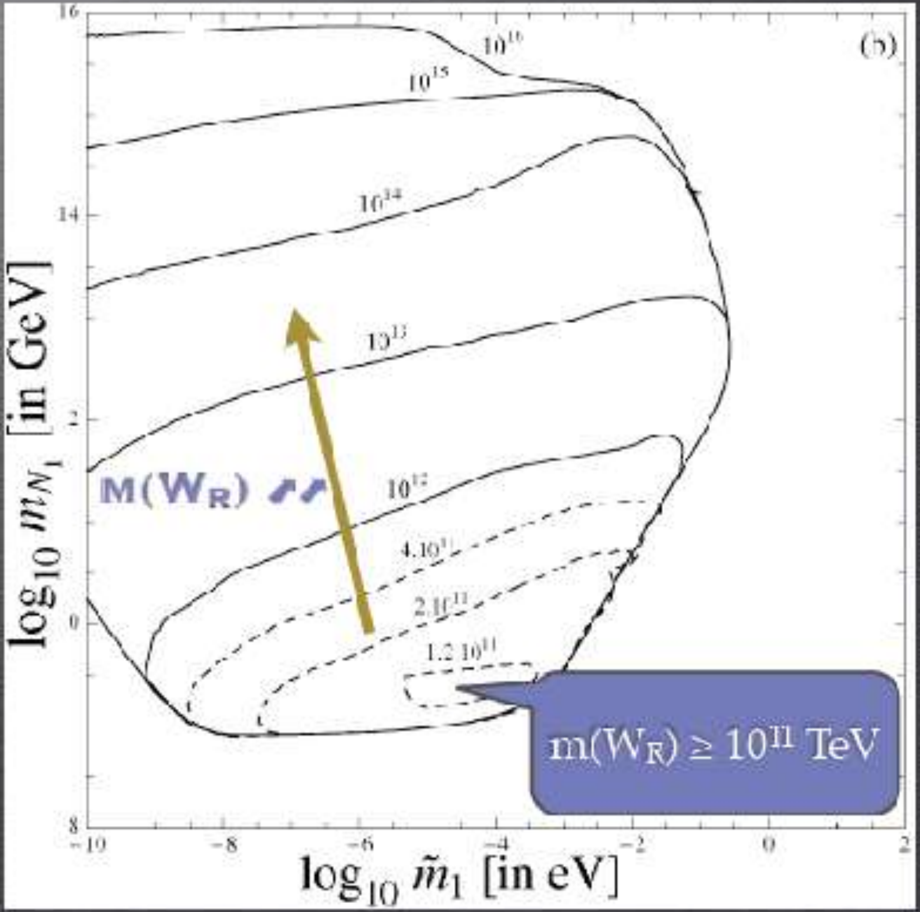
For $\varepsilon_{CP} = 1$



m(W_R) ≥ 18 TeV

M(W_R) ↗↗

For $\varepsilon_{CP} = \varepsilon_{DI}$



M(W_R) ↗↗

m(W_R) ≥ 10^{11} TeV

See T Hambye's talk

Leptogenesis is by far the most attractive way to generate the current baryon asymmetry,
It is extraordinarily sturdy and resilient, and almost hopeless to confirm

BUT

finding a W_R at a collider near you would kill at least the « type 1 » leptogenesis (= through asymmetrical N decay)

probably the only realistic way to EXCLUDE simple leptogenesis !

Neutrinos masses in the Standard Model .. And a bit beyond…

The simplest…
Just treat them like other fermions,

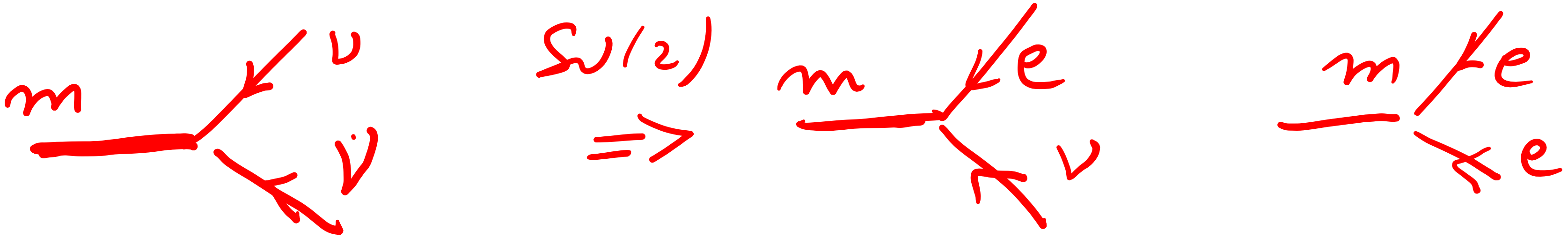
Introduce ν_R and a Yukawa coupling λ

$\lambda < m_\nu / m_W < 10^{-11}$
A bit inelegant, but there are other large/small Yukawa ratios in the SM (top/ electron = 3 $10^{5)}$

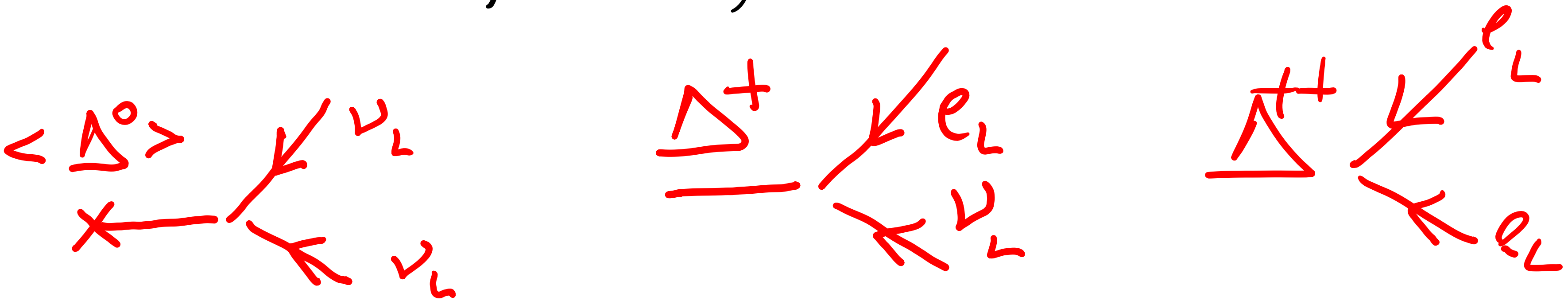
In this context, the ν_R is all but unobservable, as its sole role is in giving mass .

We can also try to do without the ν_R, and use a Majorana mass for the sole ν_L

-- But such a term breaks SU(2) invariance, and we would need a scalar triplet, with a vev through spontaneous symmetry breaking.



$\Rightarrow \quad \Delta^{++}, \quad \Delta^{+}, \quad \Delta^{0}$



Such a breaking V_L would upset the mass ratio W/Z

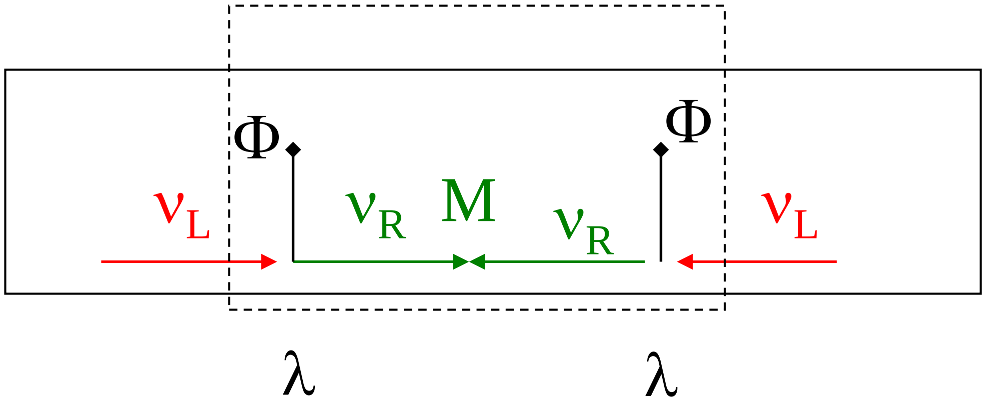
But is acceptable if small enough, for instance ..

$$\langle \Delta^0 \rangle = v_L < v/100 \qquad \frac{g v}{2} = m_W$$

This solution is not more costly in terms of « degrees of freedom » than the introduction of right – handed neutrinos, … it deserves study at the LHC

A poor man’s triplet

We can build an « effective triplet » from the Standard Model doublet, and, right-handed neutrinos ..



After diagonalization,
2 Weyl spinors
SU(2) imposes $M_1 = 0$
For $m= \lambda\, v \ll M_2 = M$ we get

$$|m_1| \approx m/M^2$$

$$|m_2| \approx M$$

$$\begin{array}{c|cc} & \xi_{Li} & \epsilon_{ik}\eta^{+}_{Rk} \\ \hline \epsilon_{il}\xi_{Ll} & M_1 & m \\ \eta^{+}_{Ri} & m & M_2 \end{array}$$

$$\lambda_1 \approx \xi_L - m/M \;\; \epsilon \cdot \eta^{+}_{R}$$

$$\lambda_2 \approx \eta_R + m/M \;\; \epsilon \cdot \xi^{+}_{L}$$

$$\lambda_1 \approx \xi_L - m/M \; \epsilon \cdot \eta_R^+$$

$$\lambda_2 \approx \eta_R + m/M \; \epsilon \cdot \xi_L^+$$

We end up with something close to a low Majorana mass left-handed neutrino, In principle, such schemes could be differentiated from the triplet by the small admixture of the R mode , which leads to a departure from unitarity in the mixing matrix .. However such effects are of order m/M and thus unobervable.

Some models may make this presence detectable, they tend however to be quite artificial … for instance :

« Double see-saw »

2 singlets

$$M_\nu = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{ccc} \nu_L & \nu_R & \nu_S \\ \hline 0 & m & 0 \\ m^T & 0 & M^T \\ 0 & M & m_\sigma \end{array}$$

$m = \lambda$ v

λ can then be large, and lead to observable effets, since the light neutrino mass is proportional to m_σ

$$m_{\nu_1} \approx (m/M)^2 m_\sigma, \qquad m_{\nu_{2,3}} \approx M \pm m_\sigma/2,$$

(remark : this is an example of « pseudo-Dirac »,

since $\nu_R + \nu_S$ act as a Dirac pair, whose contributions to the light neutrino compensate.

(an old idea, .. Langacker, Mohapatra, Antoniadis, 1986-88, jmf+Liu, recently revived…)

Mass models

Many attempts have been made at « predicting » or more often « postdicting » quark and lepton masses.

A frequent approach is based on « textures » , for instance imposing a certain number of vanishing elements in the mass matrices (hopefully in a basis-independent way), possibly via discrete symmetries (A3, A4,..)
Most have failed. (and nobody predicted the top quark in non-suspect time).

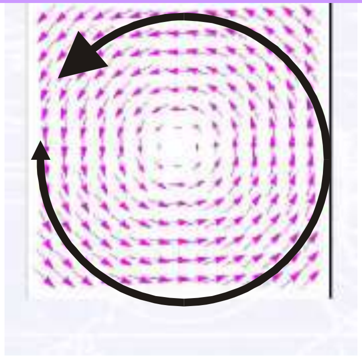
A model inspired from extra dimensions

3+1 +2 dim

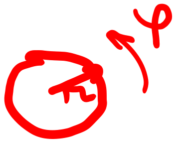
1family in 6D → 3 families in 4D

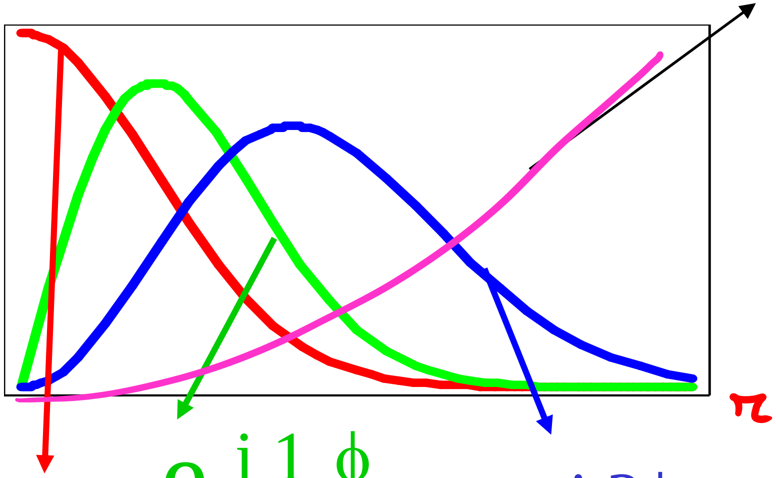
Vortex with winding number n localizes n chiral massless fermion modes in 3+1

Vortex Profile $e^{i\,3\,\phi}$



$$\Phi = e^{i\,n\varphi}$$





$e^{i\,1\,\phi}$

$e^{i\,2\phi}$

$e^{i\,0\,\phi}$

The 3 fermion modes have different shapes in r, and different winding properties in the extra dimension variable ϕ

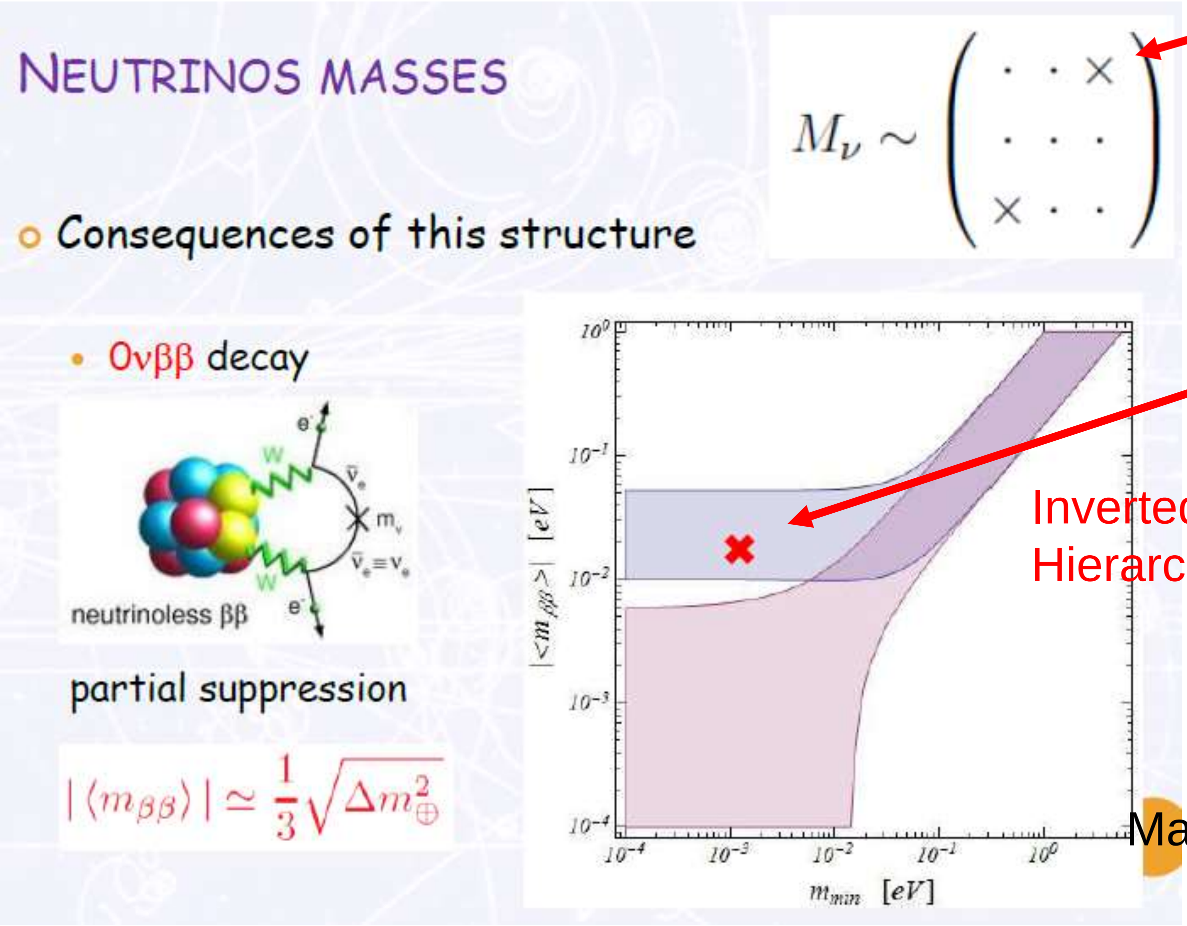
Generic prediction (quarks) :

- nearly diagonal mass matrices
- Strong hierarchy of masses linked to the overlaps at the origin

Generic prediction (neutrinos) :

- **large mixings,**
- **inverted hierarchy**
- **suppressed neutrinoless double beta decay**

Generic prediction : large mixings,
inverted hierarchy
suppressed neutrinoless double beta decay



NEUTRINOS MASSES

$$M_\nu \sim \begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \end{pmatrix}$$

- Consequences of this structure
  - 0νββ decay

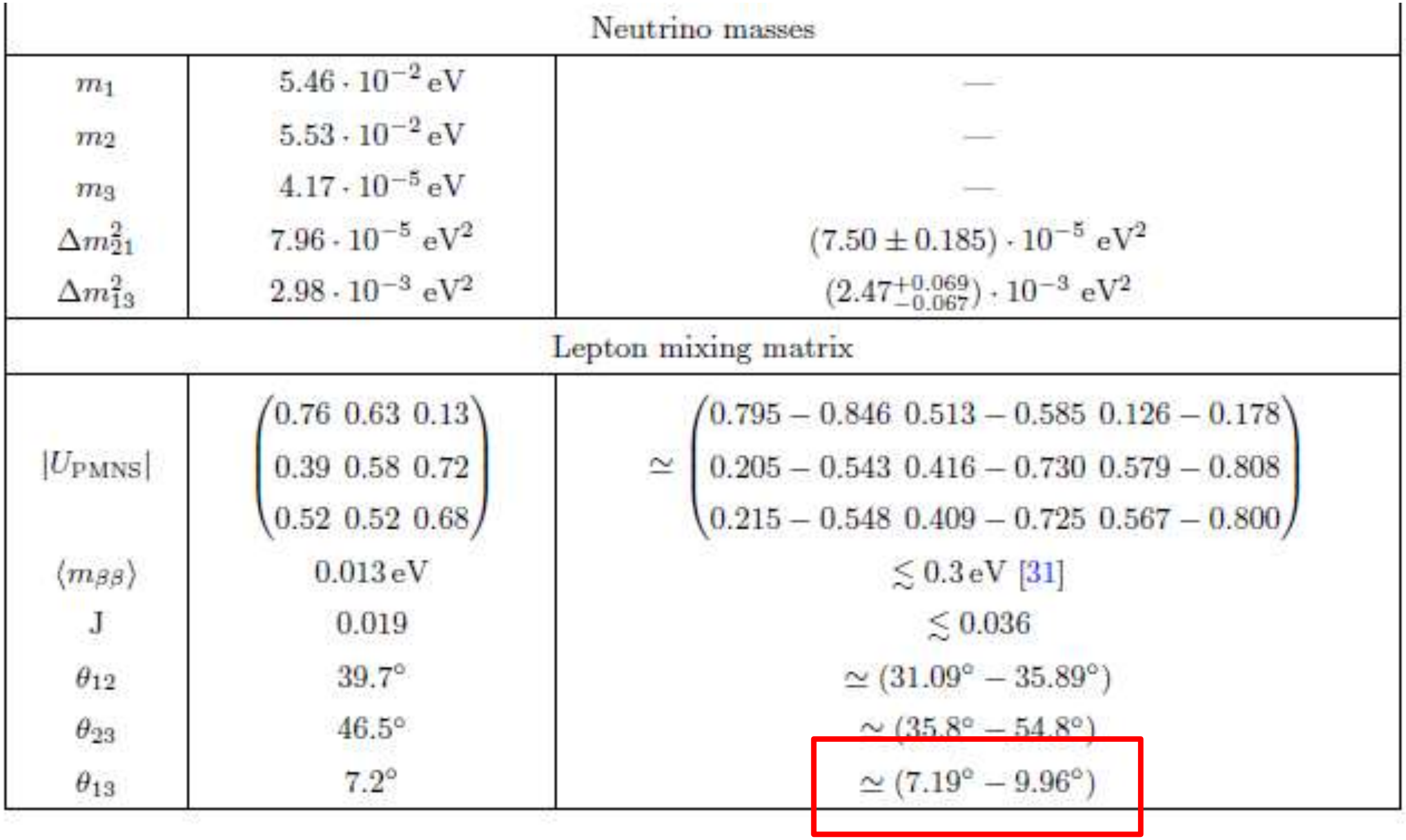
partial suppression

$$|\langle m_{\beta\beta}\rangle| \simeq \frac{1}{3}\sqrt{\Delta m^2_\oplus}$$

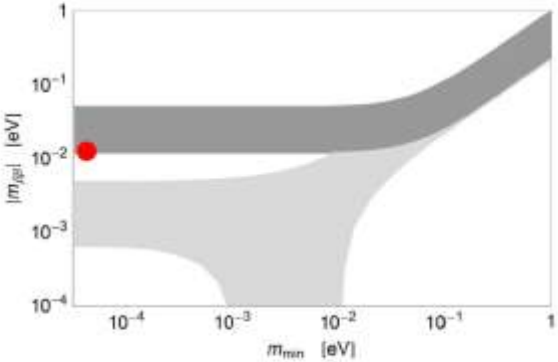
Automatically get

Inverted Hierarchy

Mass scale



	Neutrino masses	
m_1	$5.46 \cdot 10^{-2}\,\mathrm{eV}$	—
m_2	$5.53 \cdot 10^{-2}\,\mathrm{eV}$	—
m_3	$4.17 \cdot 10^{-5}\,\mathrm{eV}$	—
Δm^2_{21}	$7.96 \cdot 10^{-5}\ \mathrm{eV}^2$	$(7.50 \pm 0.185) \cdot 10^{-5}\ \mathrm{eV}^2$
Δm^2_{13}	$2.98 \cdot 10^{-3}\ \mathrm{eV}^2$	$(2.47^{+0.069}_{-0.067}) \cdot 10^{-3}\ \mathrm{eV}^2$
	Lepton mixing matrix	
$\lvert U_{\mathrm{PMNS}} \rvert$	$\begin{pmatrix} 0.76 & 0.63 & 0.13 \\ 0.39 & 0.58 & 0.72 \\ 0.52 & 0.52 & 0.68 \end{pmatrix}$	$\simeq \begin{pmatrix} 0.795-0.846 & 0.513-0.585 & 0.126-0.178 \\ 0.205-0.543 & 0.416-0.730 & 0.579-0.808 \\ 0.215-0.548 & 0.409-0.725 & 0.567-0.800 \end{pmatrix}$
$\langle m_{\beta\beta} \rangle$	$0.013\,\mathrm{eV}$	$\lesssim 0.3\,\mathrm{eV}$ [31]
J	0.019	$\lesssim 0.036$
θ_{12}	39.7°	$\simeq (31.09^\circ - 35.89^\circ)$
θ_{23}	46.5°	$\simeq (35.8^\circ - 54.8^\circ)$
θ_{13}	7.2°	$\simeq (7.19^\circ - 9.96^\circ)$



JMF,M Libanov, FS Ling, S Mollet, S Troitsky

Note a non-vanishing θ_{13} was predicted (in previous version) *before observation*